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ABSTRACT

Seven concept papers are presented that focus on the changes needed in the mathematics curriculum, K-12, to make mathematics teaching in Oregon school districts more relevant to the mathematics used in today's technological society. Task forces developed the papers, which addressed the major issues of reform in the mathematics curriculum: elementary school mathematics, middle school mathematics, geometry, algebra, statistics and probability, meeting graduation requirements, and vocational and technical mathematics. Two strands of thought are common to all seven papers: mathematics should be taught with a "hands-on" approach, and mathematics disciplines should be incorporated as content strands throughout the curriculum. For each paper, a brief rationale is given, followed by a summary. In the papers, curriculum objectives are delineated and teaching methodology is discussed. In some, specific activities are also described. (MNS)

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PREFACE

The Oregon Mathematics Education Council (OMEC) launched a mathematics curriculum project in May 1984. Nearly 100 enthusiastic, dedicated volunteers, a majority of whom are classroom teachers, worked on the task forces of the project. These groups worked to develop a set of concept papers relating to major issues of reform in mathematics education. Their ideas are shared now as a set of The Oregon Mathematics Concept Papers. They are available as a complete set or as separate papers. You are encouraged to read those papers of special interest and to discuss the ideas with colleagues. Duplicating the papers for such noncommercial purposes is encouraged.

OMEC is a widely representative advisory group, representing teachers at all school levels, counselors, administrators, school boards, business, labor, industry, and parents. The Oregon Mathematics Concept Papers are being disseminated by the Oregon Mathematics Project, working with a number of mathematics organizations, and many individuals. The organizations are:

Oregon Mathematics Education Council
Oregon Council of Teachers of Mathematics
Oregon Math Leaders
Teachers of Teachers of Mathematics
Community College Math Chairers
Honors Institutes

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The Oregon Department of Education supports, facilitates and coordinates ongoing efforts among these organizations, including the OMEC Curriculum Project. Don Fineran serves as Department liaison to all of the above organizations.

The OMEC would like to thank Al Elkins of Porter Public Relations, Portland, for his help in editing these papers.

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INTRODUCTION

Reform and excellence have become synonymous with educational change in the 1980s as legislators, parents, school administrators, boards of education, and business leaders met to discuss the state of education in their cities and states. Many may have left those discussions thinking their concerns fell on deaf ears and no recommendations for change would occur.

This report proves those skeptics wrong. The proof can be seen in seven concept papers that focus on the changes needed in the entire mathematics curriculum, K-12, to make mathematics teaching in Oregon school districts more relevant to the math we use in today's technological society.

What served as the impetus for the creation of these seven papers? One factor was the hue and cry of society for educational change. This was exemplified by the work done by eight citizen advisory committees, who were called into action by State Superintendent of Public Instruction, Verne A. Duncan. The committee reports served as a basis for the development of "The Oregon Action Plan for Excellence." This plan points the way to making good educational programs in Oregon better. As a result of this plan, the Oregon Department of Education created Essential Learning Skills, a document which helped define the learning skills for elementary and secondary education. This was the first step in the development of statewide curriculum.

Another factor was the strong desire within Oregon's mathematics education community to improve mathematics teaching. Task forces were formed, through the Oregon Mathematics Education Council, to consider major areas of concern in mathematics teaching. After spending countless hours reviewing the literature, studying research findings, participating in discussions, debates, and conferences, the OMEC task forces developed papers that address the major issues of reform in mathematics curriculum in Elementary School Mathematics; Middle School Mathematics; Geometry; Algebra; Statistics and Probability; Meeting Graduation Requirements; and Vocational and Technical Mathematics.

This report is a compilation of all these concept papers. While each paper is autonomous, there appear to be two strands of thought that are common to all of the papers:

1. Mathematics should be taught with a "hands-on" approach. New concepts are introduced at the concrete stage, then students gradually move on to a semi-concrete stage, and finally can apply related skills at the abstract level.
2. Mathematics disciplines, instead of being fragmented by either ages or grades, should be incorporated as content strands throughout the entire K-12 academic experience.

It is also interesting to note that, specifically in the high school curriculum area, this point appears in many of the papers:

> In high schools, alternatives must be offered to the traditional mathematics of Algebra I-II, Geometry, Analysis, and Calculus. More importantly, alternatives must include skills at the concrete and semi-concrete levels, as well as the abstract.

These seven papers bring us a step closer to the next phase in our statewide curriculum reform: the development of the curriculum that will be common across districts and that will become required statewide. As a matter of fact, the Oregon Department of Education is currently studying these papers as a major source of ideas for statewide mathematics development.

RATIONALE AND SUMMARIES

The seven papers' titles are listed below, with a rationale and summary statement for each. The rationale statement describes the reasoning behind each paper's development. The summary is a brief description of how the paper addresses the subject.

I. Concept Paper No. 1: Elementary School Mathematics, K-5

Rationale: The task force feels that mathematics is fun and should be taught to children with that in mind. Children learn best when they deal with the concrete, rather than the abstract; when they have objects to manipulate, rather than sums to memorize; when they are taught to think rather than told what to learn.

These recommendations, if followed, will provide a mathematics curriculum which encourages children to think about how to solve a problem; how to estimate (and how to decide when estimation is appropriate); how to decide whether to solve a problem with manipulations, with pencil and paper, or with a calculator or computer; and how to judge the reasonableness of an answer. The task force believes that these skills will give children a better understanding of mathematics and a broader base on which to build toward understanding abstract mathematics.

Summary: The curricular outline, in graph form with introductory text, is divided into four broad categories. They are Computation and Place Value; Measurement; Geometry and Visualization Skills; and Statistics and Probability.

Key elements of the suggested revision are:

1. Greater use of manipulative materials.
2. Decreased emphasis on paper and pencil computation, and increased attention to mental arithmetic, approximation, and estimation.
3. Less emphasis on computation with fractions, especially the difficult ones.
4. Greater use of calculators and computers.
5. Increased (and earlier) study of geometry concepts and of decimals.
6. Earlier introduction of the concepts of statistics and probability, taught with concrete representations.
7. Greater emphasis on the problem solving approach in mathematics education, and a related emphasis on real-world applications.

The task force makes recommendations about tests and texts that support a learner-active, developmental approach to teaching mathematics. The section contains specific recommendations for both methods and materials.

II. Concept Paper No. 2: Middle School Mathematics, 6-8

Rationale: Oregon graduation requirements for mathematics have increased; changes in the role of middle schools have been made: computers and calculators have a greater part in students' lives.

All these changes are addressed by this task force. Calculators and computers raise questions for math educators: How large a role should they play? When is their use appropriate? If a math class teaches only repetitive computation, without explaining larger concepts, can a calculator be permitted at all? The paper addresses these questions, and more.

The task force has devised goals for middle school mathematics education that are consistent with the needs of our technological society. Their recommendations reflect research findings on how children learn math. They have produced a curriculum which teaches a problem solving approach; viewing mathematics as relevant, applicable, and functional outside the classroom, rather than abstract, theoretical, and academic.

Summary: The task force describes its recommended curriculum in terms of nine "Instructional Themes," such as Problem Solving, Applications, Developing Concepts, and Relational Thinking. These themes are developed with seven "Content Strands," (examples are Problem Solving Skills and Strategies, Computational Skills and Measurement).

The task force believes that developing the themes in this way will help students make the transition from the concrete to the abstract more smoothly and will help them to integrate knowledge into all areas ("strands"). Sections I and II list and describe the themes and strands.

In Section III, the task force expresses concerns about the way personal finance is taught, particularly to those students who will not complete school; self-paced instruction; vertical acceleration and enrichment practices; assistance for low-achieving students; middle school algebra courses; effective instruction (active teaching) and relevance; staff development, inservice training, and textbook use; and recommends specific teacher and pupil materials.

A final section discusses testing methods, cooperative study, the uses of computers, and possible course sequences for students, depending on their abilities and interests (i.e., college, vocation, etc.).

III. Concept Paper No. 3: Geometry, K-12

Rationale: The task force believes that all students can benefit from learning geometric problem solving, concepts, and applications. Geometry is seen as a unifying theme in mathematics, rather than a separate discrete discipline. As such, it can aid in the teaching and understanding of arithmetic, measurement, and algebraic concepts.

Since complete understanding may take a long time to attain, the task force believes that concepts should be introduced early--even in kindergarten--and integrated into mathematics courses throughout all grades. Geometry skills enhance students' abilities to visualize solutions, providing students with a tool they can use every day, in many situations.

Summary: This paper provides specific suggestions, including materials for integrating the study of geometry into all grade levels. At the lower grade levels, particularly, this is an informal, applied form of geometry; the more abstract, proof-oriented material is introduced much later. The concepts introduced in the primary grades are expanded and refined as the student progresses. In the "topics" sections, the concepts involved are discussed, but the curriculum's emphasis remains on problem solving and applications. The study of formal deduction is reserved for only the ablest secondary students.

IV. Concept Paper No. 4: Statistics and Probability, K-12

Rationale: The task force finds that studying statistics and probability has many advantages for students. Among these advantages are: using statistics to make real-world decisions; developing critical thinking skills and ways to cope with uncertainty; and thinking more consistently, followed by justifying their thinking with numerical information.

Since so much information is presented in terms of probability, it is essential that schools teach students to understand what probability truly means. With the introduction of computers in the classroom, teachers can now demonstrate probability in many exciting ways.

In this paper, hands-on activities and student-controlled experiments are described which provide students with meaningful, exciting explorations into statistics and probability.

Summary: This paper explores the various levels of learning in statistics and probability, from simple hands-on activities for kindergarten to data analysis for high school students. It details a K-12 course of study, with specific learning goals for each grade level.

The focus is on applicability, real-world relevance, and student involvement at all levels. The effective use of computers is addressed. An illustrative activity is described and expanded through all grade levels.

V. Concept Paper No. 5: An Integrated Mathematics Curriculum for High School

Rationale: The task force reviewed past and current reports, and found a major theme calling for a unified approach to teaching mathematics. In such an approach, we see each topic contributing toward a growing organization of related information.

In 1923, The Mathematics Association of American, in a report titled The Reorganization of Mathematics in Secondary Education, recommended a program of unified courses that would help students grasp interrelationships between arithmetic, algebra, and geometry. A second report, in 1940, expanded on the same theme, adding that students needed to be taught problem solving, not merely abstract drills.

Many of these ideas have been incorporated into middle school mathematics curriculums. The task force is recommending their extension into high school courses and has developed a plan for doing so. They believe the curriculum should be: (1) responsive to the changing needs of the students, (2) accepting of the use of calculators and computers, and (3) sensitive to the difficulties students have with abstract concepts and repetitive drills at the abstract level.

Summary: After discussing in detail what has gone before in algebra instruction, the task force details: (1) themes in mathematics instruction, (2) basic concepts, skills, and techniques to be taught, (3) generalization about a unified high school mathematics course, and (4) concerns and issues.

The "Basic Concepts" section is a technical, extensive outline of coursework for an integrated program. In the final section, "Concerns, Issues, and Points of View," the need for a unified, integrative approach is discussed. The role of computers is explored, especially as they force a shift of emphasis from symbol crunching and equation solving (computation) to concepts and applications. Finally, recommendations are made for inservice for instructors.

VI. Concept Paper No. 6: Oregon Graduation Requirements: Mathematics

Rationale: The fact that a second unit of mathematics is now required for graduation provides a strong incentive to develop new sequences at the high school level. The traditional college preparatory track for college-bound, math-oriented students is not addressed in this paper. The task force has, instead, identified three populations of students for whom that track is not appropriate: those requiring special education; those with behavioral or basic mathematics problems (the "basic" or "general" math student); and those who lack the prerequisites for studying the theoretical abstract track.

The task force takes the position that assuming these three populations of students can satisfy a second math unit from the traditional mathematics curriculum will lead to disaster for the students. In this paper, they recommend courses with realistic expectations for these students.

Summary: This paper describes alternative mathematics courses for the first and second years, for noncollege-bound students. One of the goals is to improve the students' confidence in their mathematical skills. With this in mind, the task force describes several topics for the

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introductory unit: whole number patterns, logic puzzles, and activities which apply problem solving techniques. Paper and pencil drill is de-emphasized in favor of problem solving and concept development.

For the second unit, the task force describes a course for students of lower ability or lower interest level. This course is designed to excite the students' interest, with practical uses of mathematics concepts in business, sports, hobbies, travel, and consumer matters.

Options for students who have completed Algebra I, but are not interested in following the college prep pattern of Geometry and Algebra II, are also presented. The variations listed offer a range of mathematics applications.

VII. Concept Paper No. 7: Vocational and Technical Mathematics

Rationale: The task force identifies and describes two very different groups of students who are likely to take these courses. Their needs are addressed separately, since their skill levels and ultimate goals may be very different. For instance, the technical mathematics student probably has a good grasp of algebra and geometry, may be college-bound, and needs review and practical application of what has already been learned.

The vocational student, on the other hand, may need much new material, may need a course to fill the second year requirement, and may be very likely to do poorly in a course emphasizing computation and drill. The course described for the vocational student emphasizes the use of mathematics, rather than mathematics itself.

The need for courses which prepare students for the vocations, with practical information and useful skills, becomes clear when the results of the task force's survey are studied.

Summary: Vocational and technical math courses are organized with seven major topic headings, or "strands." These are Measurement, Communication, Computation, Basic Algebra Skills, Probability and Statistics, Basic Geometry Skills, and Reasoning.

In the vocational course, application, demonstration, and use will be the focus; concepts will be introduced as "techniques" or "skills."

The course may be perceived as a "survival course" for students with little interest in formal mathematics. The technical course, on the other hand, has been developed for students whose mathematics background has been fairly strong; it is presented as an alternative to senior math, precalculus, or similar courses. Practical application of concepts already mastered is emphasized.

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Oregon Mathematics Concept Paper No. 1

ELEMENTARY MATHEMATICS CURRICULUM, K-5

PREFACE

The Oregon Mathematics Education Council (OMEC) launched a mathematics curriculum project in May 1984. Nearly 100 enthusiastic, dedicated volunteers, a majority of whom are classroom teachers, worked on the task forces of the project. These groups worked to develop a set of concept papers relating to major issues of reform in mathematics education. Their ideas are shared now as a set of The Oregon Mathematics Concept Papers. They are available as a complete set or as separate papers. You are encouraged to read those papers of special interest and to discuss the ideas with colleagues. Duplicating the papers for such noncommercial purposes is encouraged.

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The OMEC would like to thank Al Elkins of Porter Public Relations, Portland, for his help in editing these papers.

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INTRODUCTION

The OMEC Task Force on elementary school mathematics curriculum has taken seriously its charge to recommend a curriculum that will be suited for the future. The phrase "turn of the century" suggests not looking backward but forward to the year 2000 and beyond. While the task force members did not pretend to know precisely what mathematics education would look like at the turn of the century, they did use their reading and study, and their best knowledge and teaching experiences, to design a curriculum for children. The recommendations made in this paper incorporate principles of child development, successful teaching practices, literature on mathematics education, and many hours of discussion.

The members of the task force are teachers. Their points of view regarding mathematics education were constantly tempered with their knowledge of children, how children behave in the classroom, how children learn best, and the mathematics skills most used beyond school. Early in this work, it was recognized that success plays an important role in learning mathematics. Teachers should do their best to assure that students will have successful experiences as they learn mathematics. It was also agreed that mathematics is and can be fun. Teachers should help children capture the pleasure and excitement of mathematics. Children need to see mathematics as real and meaningful to their early lives, as well as useful in the future.

Highlights of the task force deliberations include:

1. Change in the elementary curriculum:

    a. More extensive study of geometry.
    b. Increased integration of mathematical "strands" as opposed to segregated topics.
    c. Less time spent on little-used computation such as long division and operations with fractions.

2. Broadening the instructional tools and roles of the teacher:

    a. Use calculators, mental arithmetic, manipulatives, paper and pencil, and computers when appropriate.
    b. Increase use of manipulative materials to serve as models for learning.
    c. Increase flexibility in using the mathematics textbook.
    d. Recognize that the textbook, by its nature, cannot be a comprehensive curriculum.
    e. Use standardized tests to assist in evaluating the curriculum. They should not determine the curriculum.
    f. Become decision makers with the authority to implement decisions.
    g. Participate in workshops, conferences, and courses designed to provide innovative ideas about teaching mathematics.

The members of the task force question the total reliance on the textbook approach toward teaching mathematics. Children that receive only a textbook approach are being cheated. Other approaches to teaching mathematics deserve more emphasis. Teachers should help children develop concepts which underlie basic mathematical skills and provide active experiences in which learning can take place. Examples can be found in Mathematics Their Way (Addison-Wesley) and Problem Solving in Mathematics (Dale Seymour) among others.

The members of the task force are concerned about the importance placed upon standardized test results. The curriculum has become a preparation for taking standardized tests which are designed primarily from the content of mathematics textbooks. For example, long division and operations with fractions survive within the curriculum, in part, because traditionally they have been included in the tests. Thus, close adherence to the textbook in preparation for taking an exam has limited the opportunities for children and teachers to reach their fullest potential as learners and teachers. There should be a continued evolution of standardized tests to assess the knowledge of turn of the century mathematics.

Research has proven that mathematical concepts can best be introduced to children through the use of manipulatives at all levels. At the earliest stages children manipulate, count, add, and subtract with objects. They learn a great deal about number relationships without written symbols. At the representational stage, children make the connection between symbols and numerical concepts. Most of the teaching time should be spent at the concrete and representational levels.

Teachers must decide when to digress from the textbook, bypass chapters or sections of the textbook, and use more effective and illustrative strategies. Careful consideration must be given to which mathematical skills children need in order to function successfully at each stage in their development. Teachers can best judge the needs of the children in their own classrooms. If a lesson is not accomplishing what teachers feel it should, they must trust their knowledge and judgment to modify lessons so the children can be successful. It is time for them to consider the children's needs and speak out against skills that are taught at specific grade levels and inappropriately placed on tests.

In planning instruction that will appropriately challenge all youngsters, teachers often find themselves exploring questions pertaining to vertical acceleration or enrichment. The task force recommends the position taken by the National Council of Teachers of Mathematics, in April of 1983, in which they state:

> That vertical acceleration be considered only for a limited number of highly talented and mathematically creative students whose interest and attitudes clearly indicate that they have the ability and perseverance to complete a carefully designed sequential curriculum. For all but this select group, a strong expanded program emphasizing mathematics enrichment is preferable.
>
> Official NCTM position, April 1983

The curriculum described in this paper lends itself readily to the enrichment approach recommended by the NCTM. The above statement and a discussion on the acceleration/enrichment question are included in an OMEC unpublished paper, available from Don Fineran, Oregon Department of Education. Although the paper is secondary oriented, the concept and rationale discussed in it suggest some ideas for the elementary school program.

The members of the task force support a continued thrust in problem solving throughout the mathematics curriculum. Problem solving is an approach to thinking about mathematics and belongs at every stage and in every topic presented in the mathematics program. Children should share and refine their thinking processes. They should estimate whenever practical. They should experience open-ended questions and activities. They should become skillful in attacking problems.

School districts should provide professional growth experiences for their teachers. Teachers who have used only a textbook approach must have inservice training to develop skills to enable them to provide more versatile instruction. Districts should provide ongoing support for teachers to attend conferences and workshops. Opportunities should be provided for teachers to observe master teachers demonstrating exemplary mathematics instruction in elementary classrooms.

It is with these thoughts in mind and in the spirit of sharing that the task force presents the following curricular outline.

COMPUTATION AND PLACE VALUE

Computation and place value will continue to play an important role in the elementary mathematics program. The emphasis should be on concepts and skills that children in kindergarten through fifth grade will need. In the coming years, those students will be faced with technology advancing at an incredible rate. They will need to be able to select the most appropriate method of computation. Alternative methods may involve manipulation of concrete objects, mental arithmetic, or calculators and computers. Often paper and pencil will still be appropriate, although teachers need to de-emphasize that mode and challenge students to choose other methods more fitting to the particular problems. It is important to make math useful to students, both now and when they become adults.

Throughout the math curriculum, and especially in computation and place value areas, understanding the concepts should be a major concern. To develop these concepts at all grade levels, it is vital to use a broad range of manipulatives and to gradually move from a concrete level to a representational level and finally to an abstract level.

Not every problem requires an exact answer and there are many times when an approximation is appropriate. Students need to be able to make decisions as to when an approximation is sufficient or if an exact answer is needed. They need to be able to judge the reasonableness of answers resulting from any method of computation.

Certain topics, traditional in elementary math instruction, need to be de-emphasized: long division, computation of fractions, multiplication of large numbers. Increased emphasis should be placed on estimation, sensing reasonableness of answers, problem solving, measurement, geometry, probability, developing concepts by use of manipulatives, and choosing appropriate tools to solve problems.

COMPUTATION & PLACE VALUE

K	1	2	3	4	5
Rote count to 20. →	→	→ 100			
Use number patterns to facilitate skip counting to 100 by 5's.	Skip count to 100 by 2, 5, 10.	Skip count by 3's.			
	Use number patterns to	→	"count on" to 100.		
Estimate and determine quantity by counting objects up to 20. →	→	→	→ 100		
Count backwards from ten using objects. →	→ 20				
Recognize at a glance 2, 3, 4 objects arranged in varying patterns.					
Recognize and show meaning of numerals zero to 10 out of sequence. →	→ 0–20				
	Write numeral given word or number of objects.				
	Write numerals 1 - 100.	→			
	Recognize, generate and extend number patterns.		→	→	→
Demonstrate with objects equal/not equal more/less. →	→	Read, write, compare and order numbers to 100.	Read, write, compare and order numbers to 10,000. →	→	→

K	1	2	3	4	5
Use objects to show place value through grouping and exchange. →	→	Demonstrate 2 digit place value with concrete models.	3-digit. →	→	→
Work with the process of addition and subtraction using objects less than 10. →	→				
	Use manipulatives to interpret a written + or − sentence (up to 10). →	→			
	Mentally add or subtract 10 to or from a single number.	Mentally add or subtract multiples of 10 to or from a number. →	→	→	→
		Acquire strategies to aid in recall of all basic add/sub facts: e.g., counting on, zero, properties, doubles and neighbors. →	→	→	
		Solve 2 digit + or − problems using manipulatives.	3 digit problems. →	→	
Calculators available for exploration or appropriate calculations. →	→	→	→	→	→
			Read, write, compare order, using models, fractions (1/2, 1/3, 1/4) and decimals in tenths and hundredths. →	→	Thousandths
Use concrete models (area, volume, quantity, length) to understand halves and wholes. →	→	Halves, wholes, fourths, thirds.	Use concrete models to understand commonly used fractional parts (1/2, 1/3, 2/3, 1/4, 3/4, 1/10, 5/10). →	→	→
			Represent tenths and hundredths using concrete models. →	→	Thousandths

K	1	2	3	4	5
Engage in informal experiences with multiplication and division.	→				
		Use concrete materials to interpret multiplication/division: e.g., area model, set models, arrays.	→		
			Use strategies to aid in recall of basic multiplication/division facts, i.e., doubles, nines, count by 2, 5's, commutative and associative properties.	→	→
			Solve 2 by 1 multiplication problems.	→	→
				Solve 2 by 2 multiplication problems.	→
				Solve division problems with 1 digit divisors.	→
		Estimate approximate answers.	→	→	→
		Check for reasonableness of answers by estimating products and quotients.	→	→	→
		Choose appropriate method to solve multiplication and division problems, i.e., calculators, mental, manipulatives, paper/pencil, computer.	→	→	→

K	1	2	3	4	5
	Demonstrate that the "size" of a fraction depends upon the size of the units (Example one half of a small box is smaller than one half of a large box.).		Read and write commonly used fractions. →	→	→
		Approximate fractions above. →	→	→	→
			Use concrete models to demonstrate equivalent fractions. →	→	→
			Use concrete models to demonstrate add/sub of fractions with like denominators. →	→	→
				Use concrete and pictorial models and practical applications to illustrate commonly used fractions, mixed numbers and equivalent fractions, e.g., 2/3, 5/8, 1 1/2.	
			Use concrete models to understand decimals to hundredths. →	→	Thousandths
			Read, write, order and compare decimals using concrete materials to hundredths. →	→	Thousandths
			Use decimal notation to add and subtract amounts of money.		
				Addition and subtraction of decimals. →	→
Informal →	→	Practice with various methods to solve problems--then select: i.e., manipulation, mental, calculator, paper/pencil.	→	→	→
Informal →	→	Check for reasonableness of answers with whole numbers, fractions and decimals.	→	→	→

K	1	2	3	4	5
		Estimate and approximate answers to whole number, fraction and decimal problems.	→	→	→
					Multiplication of decimals by whole numbers using realistic situations (money).
Solve simple problems presented verbally by role playing and by use of manipulation.	→				
	Solve + or - word problems.	→	→	→	→
	Create + or - word problems to match number sentence.	→	→	→	→
		Solve problems with multiple correct answers.	→	→	→
		Recognize problems which cannot be solved because of too little information.	→	→	→
		Use guess and check for number problems.	→	→	→
			Solve word problems including money, measurement data tables, charts, graphs.	→	→

MEASUREMENT

Children must use objects and models in hands-on activities measuring and comparing to comprehend measurement concepts. This enables a child to apply the knowledge gained at a concrete level to more abstract situations. It takes many hands-on experiences through all of the elementary school years to develop a good understanding of measurement.

Making comparisons and using models of non-standard units to determine a measure are important steps in a child's early mathematical understanding. Estimation must be a key factor in children's first measurement experiences. As children mature they should become familiar with both metric and traditional systems.

To accomplish these broad skills, it is important to provide an environment which allows children to explore physically the objects around them. As children interact with the world, they will internalize and understand measurement systems.

MEASUREMENT

Measurement

K	1	2	3	4	5
- Identify longer or shorter of two objects.	→	Recognize need for standard units of measurement.	→	→	→
- Order sets of actual objects by length.	→	→	Recognize and use meters, centimeters feet, yards, inches.	Measure to nearest 1/4 inch and centimeter.	Measure to nearest 1/16 inch and to nearest millimeter.
- Measure length using non-standard, uniform units. (A series of the objects along length.)	→	→	→	→	→
	Lining up measuring tool with item to be measured.	→	→	→	→
	Estimating length of an object	→	Select the most appropriate instrument and linear unit to measure a given length or distance.	→	→
					Convert among units of length within the same system.
- Use a balance scale to determine heavier or lighter (direct comparison).	→	→	→	→	→
	Estimate weight in non-standard units.	→	Use traditional and metric units of weight to estimate and measure common classroom objects.	→	→

K	1	2	3	4	5
- Make direct comparisons of capacity using nonstandard units.	→	→	→	→	→
			Use standard units to measure volume and capacity.	→	→
				Use practical experiences to approximate doubling and halving quantities.	→
	Estimate number of units fitting into larger container.	→	Use a set of uniform cubes to determine the capacity of a box.	→	Compute volume.
	Identify and order by value penny, nickel and dime.	→ All U.S.coins.	→	→	→
	Role play making change up to dime.	→ up to a quarter	Use coins to show equivalent amounts up to one dollar.	Use bills and coins to show equivalent amounts up to $5. Role play making change for $5 & $10.	Make change to nearest whole dollar and appropriate multiples of $5. (e.g., give change for $17.98 purchase from $20.)
		informal ← rounding off dollars and cents		→	→
- Demonstrate an understanding of time (day, week, month, year). Introduce today, tomorrow, yesterday.	→ seasons				

Measurement

K	1	2	3	4	5
- Develop skill in estimating time duration using non-standard measures of elapsed time, e.g., Sand, water, candle-burning, ice melting, etc.	→	→	→	→	→
			Read time on digital and standard clocks.	→	→

GEOMETRY AND VISUALIZATION SKILLS

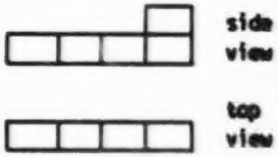
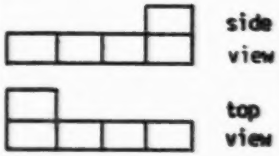
Geometry and visual thinking skills should play an expanded role in the elementary mathematics program. Increased emphasis should be placed on estimation, constructing two- and three-dimensional figures, developing an appreciation for geometric applications in the environment, and sharpening visualization skills. Children will need to be aware of geometry throughout the school year; not merely when the topic appears in the textbook.

Learning geometry should be active and a variety of many manipulative materials should be available and used. These include pattern blocks, geoboards, mira, paper models, and LOGO. It is important to provide a rich environment of exploratory and learning activities.

GEOMETRY AND VISUALIZATION SKILLS

K	1	2	3	4	5
Identify: Square, rectangle, circle, triangle (various kinds and rotations) occurring in the environment. →	Cube, cylinder, sphere. →	→	Identify properties of polygons and common geometric solids. →	→	→
	Construct 2-D (geoboards, plastic straws.	→	Make 3-D shapes from paper patterns (cut and tape) and other materials.	Looking at pattern and predict what 3-D shape it will fold into. →	→
Visualize, build, extend, identify patterns. →	→	→	→	→	→
Vocabulary: Over, under, straight, curved, open, closed, near, far, right, left.	→				
Engage in informal experiences with symmetry (pattern blocks) and in nature.	Identify symmetry - informal experiences.	Begin to create designs of symmetry.	Identify symmetry in environment and create designs with symmetry.		
	Sort 2-D and 3-D items.	→	→	→	→
				← Coordinate graphing (battle ship)	→

GEOMETRY AND VISUALIZATION SKILLS

K	1	2	3	4	5
Reproduce simple 3-D model from blocks looking at 3-D model. →	More complex. →	Construct model from picture (using pictures of models built in K-1).	← More advanced →		
			Draw cube from 3-D model, side views.	Draw top and side views of 3-D models.	Seeing picture of side views/top views student should be able to construct 3-D model.

STATISTICS AND PROBABILITY

The OMEC Elementary Task Force endorses the proposal of the OMEC Task Force on Statistics and Probability for this strand of the curriculum. For the fully detailed presentation, please refer to the complete OMEC Task Force report, "Oregon Mathematics Concept Paper No. 4: Statistics and Probability, K-12." The following are the grade level recommendations of that task force.

GOAL 1:

The learner will demonstrate ability to collect and record appropriate data in a given situation.

Students will:

K - 1	2 - 3	4 - 6		
● Gather factual data from personal experiences and environment.				
● Use physical objects to record data.		● Gather and record factual data from variety of resources. - Integrate with other subject areas.		
● Tally by 1's.	● Tally by 5's (~~1111~~ 1).	● Collect data from probability experiments.		

GOAL 2:

STATISTICS AND PROBABILITY

The learner will demonstrate ability to organize data and make accurate representation of the data in tables, charts, graphs, or diagrams.

Students will:

K - 1	2 - 3	4 - 6			
• Sort and classify people and objects.	→				
• Make real graphs which compare groups of objects.	→				
• Make picture graphs which use models to stand for real things.	→				
	• Construct bar graphs.	→			
		• Organize information into tables/charts and diagrams given appropriate scale: - Box Plots - Box-and-whiskers - Line plots - Venn diagrams			
		• Construct line graphs.			
		• Use computer generated graphs and tables.			

GOAL 3:

The learner will demonstrate ability to develop and use sampling techniques.

Students will:

K - 1	2 - 3	4 - 6		
● Take samples from classroom members and family.	→	→		
	● Take non-random samples from larger groups such as school population.	→		
		● Take random samples from the community.		
		● Understand the relationship between size of sample and degree of certainty.		
		● Understand biases involved in sampling techniques.		

GOAL 4:

The learner will demonstrate ability to use various techniques to represent possible outcomes of an experiment and assign probabilities to these outcomes.

Students will:

K - 1	2 - 3	4 - 6			
• Experience activities which have outcomes that depend on chance.	→	• Understand fair and unfair games for experimentation.			
	• Experience activities with non-chance outcomes.	→			
	• Make informal predictions - - possible/impossible - likely/unlikely - certain/uncertain	→			
		• Understand meaning of probabilities of 0 (impossible) and 1 (certain).			
		• Use charts, tables and lists to organize all possible outcomes of an experiment.			

GOAL 5:

The learner will demonstrate ability to interpret data to make comparisons, predictions, and inferences.

Students will:

K - 1	2 - 3	4 - 6		
● Read and analyze information on graphs to answer comparison questions.				
	● Pose questions which can be answered using data.			
	● Draw conclusions based on organized data.			
		● Understand the concept of and determine the arithmetic mean.		
	● Use intuition to make choices.	● Justify conclusions with intuition and reason.		

ELEMENTARY MATHEMATICS CURRICULUM

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Oregon Mathematics Concept Paper No. 2

MIDDLE SCHOOL MATHEMATICS, 6-8

PREFACE

The Oregon Mathematics Education Council (OMEC) launched a mathematics curriculum project in May 1984. Nearly 100 enthusiastic, dedicated volunteers, a majority of whom are classroom teachers, worked on the task forces of the project. These groups worked to develop a set of concept papers relating to major issues of reform in mathematics education. Their ideas are shared now as a set of The Oregon Mathematics Concept Papers. They are available as a complete set or as separate papers. You are encouraged to read those papers of special interest and to discuss the ideas with colleagues. Duplicating the papers for such noncommercial purposes is encouraged.

OMEC is a widely representative advisory group, representing teachers at all school levels, counselors, administrators, school boards, business, labor, industry, and parents. The Oregon Mathematics Concept Papers are being disseminated by the Oregon Mathematics Project, working with a number of mathematics organizations, and many individuals. The organizations are:

Oregon Mathematics Education Council
Oregon Council of Teachers of Mathematics
Oregon Math Leaders
Teachers of Teachers of Mathematics
Community College Math Chairers
Honors Institutes

For additional copies of The Oregon Mathematics Concept Papers, or information regarding speakers and/or discussion leaders, contact:

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The Oregon Department of Education supports, facilitates and coordinates ongoing efforts among these organizations, including the OMEC Curriculum Project. Don Fineran serves as Department liaison to all of the above organizations.

The OMEC would like to thank Al Elkins of Porter Public Relations, Portland, for his help in editing these papers.

OMEC TASK FORCE
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MIDDLE SCHOOL MATHEMATICS, 6-8

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INTRODUCTION

Oregon Mathematics Education Council (OMEC) is a widely representative volunteer group interested in mathematics education at all levels. In spring 1984, Gary Musser, OMEC president 1983-84, invited college and school mathematics educators to a meeting at Oregon State University to discuss the mathematics curriculum changes that should take place as a result of:

- The new three-year mathematics requirement for college entrance.
- The new two-year requirement for high school graduation.
- The shift from junior high to middle school concept of school organization.
- The general lack of emphasis upon problem solving in mathematics instruction.
- The continuing quandary as to what curriculum changes should take place as a result of calculators and computers.

The middle school task force first met in June 1984. Fifteen educators attended, representing elementary, middle, high school and college mathematics teacher educators. The purpose of this task force was to make forward-looking recommendations consistent with the significant developments that have taken place in mathematics education in the nation and with recommendations being made by other OMEC task forces. We hope our work will help the Oregon Department of Education and local school districts design mathematics programs which will help citizens meet their needs in the 21st century.

This paper, "Oregon Mathematics Concept Paper No. 2: Middle School Mathematics, 6-8," is presented in three sections:

Section I - Instructional Themes Across the Mathematics Curriculum
Section II - Middle School Mathematics: Content Strands
Section III - Concerns, Issues and Points of View

Section II, Middle School Mathematics: Content Strands, includes a listing of objectives that could be used in designing a statewide test the Oregon Department of Education may consider for all eighth grade students.

SECTION I - INSTRUCTIONAL THEMES ACROSS THE MATHEMATICS CURRICULUM

The following themes should cut across all the topics or units suggested by the mathematics abilities and topics listed in Section II.

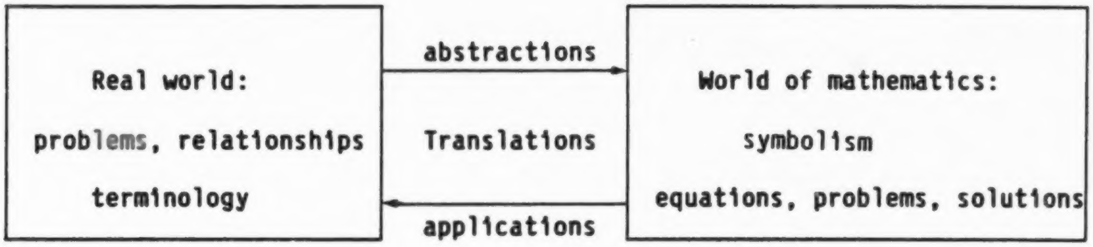
1. Problem solving

Problem solving has been designated as the central goal for mathematics in the 1980's. A problem is a perplexing situation in which an individual or group accepts the challenge of finding ways to clarify or resolve the difficulties involved. Frequently, the problem can be approached in many ways. Occasionally, the resulting investigations are nonproductive. Sometimes they are so productive as to lead to many different solutions or suggest more problems to solve.

Problem-solving skills and strategies should be explicitly emphasized and problem solving should be incorporated frequently into the approaches used in teaching the required topics for the various middle school courses.

2. Applications of mathematics

Applications should be included in all units in the mathematics curriculum. The selected applications should be consistent with students' level of development and perceived by them as real and relevant to their world. This simplified diagram suggests the relationship that should exist between mathematics instruction and the real world.



Notice that real world influences mathematics, and mathematics influences the real world. Thus, mathematics instruction must reflect not only aspects of the world and mathematics, but their interactions as well.

3. Mathematical thinking

> For many, to 'know' means to identify the artifacts of a discipline (its record). For me and many others, to 'know' mathematics is 'to do' mathematics.
>
> Thomas A. Romberg. A Common Curriculum for Mathematics. 1983 Yearbook National Society for the Study of Education, p. 121.

Romberg identifies four related activities common to all of mathematics: abstracting, inventing, proving and applying (p. 124). These activities are the natural consequences of mathematical investigations. Students should be exposed to both inductive and deductive reasoning and their roles in problem solving and in the search for relationships. Mathematical instruction should reflect that concepts can be generalized.

4. Bridging the gap between the concrete and abstract

For students, the passage between the concrete and the abstract is not so much a bridge as an entire journey. Along the way, there are many levels of abstraction, each implying a different mental image or representation. Physical objects and pictorial representation help students build these images and see the connection between the real world and abstract written symbols. Experiences with concrete representations are important to mathematics instruction, but students will not learn mathematics well with these experiences alone . It is critical that teachers consistently make explicit the linking of this appropriate concrete representation to the various images students have along the way toward the symbolic representation of any mathematical concept. In addition to going from the concrete to the abstract, students need to have experiences finding concrete representations for abstract concepts and their symbolic representations.

We still have much to learn about human growth in abstraction. As yet, there are no final answers and there seems to be no single route to abstraction. We do know that the rate at which students pass from concrete to abstract levels of thinking differs from one student to the next. Whatever the route or rate, teachers can help make passage to the abstract a smoother and more comfortable journey. (Adapted from Mark J. Driscoll, Research Within Reach, Elementary School Mathematics.)

5. Developing concepts

If students have attained a concept, they should be able to choose and recognize all examples of the concept, produce examples as well as non-examples of the concept, and at some stage interpret and produce the word or symbol for the concept. Definitions phrased at the appropriate level of abstraction can help in concept formation, but there is a danger of over-verbalization. The important thing is the idea, and rote recitation of a definition does not assure that the idea is present. Our goal should be to get students to do something which gives us feedback about their grasp of the concept. Use should be made of a variety of representational modes, especially concrete ones. These are necessary if students are to be aware of the many mental images most concepts have. The teacher's role should be to assist students in clarifying the initial mental image of the concept, to expand the concept to include more of these images, and then to exercise the necessary flexibility in the appropriate application of the concept to the task at hand. (Adapted from "The Teaching of Concepts," Mathematics Resource Project.

6. Developing the ability to make estimates

Estimation has long been recognized as a valuable, useful skill in many vocations and in daily life. With the growing use of calculators and computers, it is vital that people be able to judge the reasonableness of an answer. Also, there are many instances where an estimate is all that is possible for making an important decision. There are many ingredients necessary for helping students become proficient estimators. One of these is the development of a proper mental set for estimation. Another is the recognition that there are many ways to obtain reasonable estimates.

The above statements were adapted from an NSF Estimation project conducted by Robert E. Reys, Paul R. Trafton, Barbara Reys, and Judy Zawojewski. These statements obviously apply to computational estimation. They also are equally appropriate for the other strands listed in Section II.

7. Relational thinking

> It is precisely this fundamental problem of searching for relationships and giving accurate expression to them that mathematics has been so successful.
>
> NCTM 15th Yearbook, The Place of Mathematics in Secondary Education, p. 10.

The concept of relation is fundamental to human thought. Knowledge of relationships allows humans to go beyond the immediate and make reasonably reliable predictions of future events. The application of mathematics enables scientists and engineers to explain physical phenomenon and gain some control over it. A knowledge of probability and statistics helps us to make predictions even in the face of uncertainty.

A knowledge of mathematical relationships is necessary, but more important still is for students to develop their ability to search for relationships both in mathematics and everyday situations. This ability of relational thinking is more likely to develop if the instructional focus is as much on the search process as upon the relationships themselves.

8. Reading, writing and talking about mathematics

> Most educators agree reading in any subject matter cannot and should not be separated from comprehension and concept development in that subject, and considerable evidence shows student achievement is improved if reading is integrated with content instruction.
>
> As learners interpret and analyze their concrete experiences again and again, while relating them to oral language they hear and use at the same time, they gradually develop summarizing ideas or concepts symbolized by spoken words. Symbols growing out of learners' firsthand experiences while accompanied by oral language acquire lasting meaning.

These quotes are taken from MATH in Oregon Schools. This guide specifies that teachers should direct students' attention to the special reading and vocabulary problems of mathematical content, discuss special features of textbooks or other materials used regularly in class, give special attention to reading charts, diagrams, tables and graphs and involve students in discussing the problem solving strategies they have used. Also, since 1977 (publication date of the guide) more and more educators are beginning to accept with much justification that the process of writing, thinking and reflecting are much alike and that writing is the best way for most students to learn what they are expected to learn in school. Apparently, writing a process journal greatly aids learning. This journal might include how the student went about learning a particular topic, solving certain open-ended problems, making sense out of and applying certain concepts, and relating new ideas to previous knowledge. The recent book Thinking Mathematically, Mason, et al, 1982, is based upon the contention that writing aids learning.

Cooperative learning (Burns 1981) is another movement that has gained momentum since 1977. The movement is based upon the assumption that the transmission of knowledge is a social process which depends upon the cooperation and goodwill of all concerned. Furthermore, advocates of this movement have demonstrated that interaction is one of the essential ingredients of learning. In group explorations students share with each other and justify their thoughts about various subjects, validate facts, deal with contradictions, and rethink their own ideas. Cooperative learning reduces anxiety by encouraging communication, and as a result helps create an atmosphere in which openness and creativity can occur.

9. Using calculators and computers

> The school curriculum does not yet reflect the advent of the calculator and the now readily available microcomputer to handle complex and iterative manipulations. Problems that are actually quite simple but were unsuitable for widespread use in school mathematics because of tedious and time consuming calculations can now be included at relatively low levels of mathematics instruction.
>
> Mary Grace Kantowski

Instruction in using the calculator and computer should be integrated into the mathematics program of the middle school. Both should be used as teaching tools when this is appropriate for what is being taught.

A calculator should be used when there is an advantage for doing so. It should be used when it extends the ability to solve problems, speeds the search for patterns, eliminates repetitive, tedious and routine computations. Furthermore, it can remove the anxiety of computational failure and save time within the curriculum for other worthwhile objectives.

Mastery of basic number facts and the development of mental computations and approximation are more important than ever in this age of high technology. A careful but creative use of the calculator can be used in the mastery of these important goals also.

The computer has much potential in being used as a tool in promoting the goals of mathematics instruction. Concepts from probability, statistics, geometry, and graphing can be developed through computer-related experiences. Basic computational and estimation skills can be practiced and extended. In the process of writing programs, students can develop their own algorithms for solving many kinds of problems. Problem solving skills such as guess and check, thinking ahead, and looking back can be taught and practiced. The computer-related experiences selected should be primarily those in which pupils are in control.

SECTION II - MIDDLE SCHOOL MATHEMATICS: CONTENT STRANDS

The mathematical content for the middle school should be selected from concepts, principles, and skills under the following strands:

- Problem Solving Skills and Strategies
- Number and Numeration: Basic Concepts, Principles and Meanings
- Appropriate Computational Skills
- Two- and Three-Dimensional Geometry
- Measurement
- Probability and Statistics
- Patterns and Relationships
  Variation
  Ratio, Proportion and Percent

Calculators and computers should be used in imaginative ways for exploring, discovering and developing mathematical content from all of the strands.

Each strand and the reasons for its importance is discussed, followed by a rather complete listing of objectives that might be achieved by most students at the end of the 8th grade. The statements are written in terms of student behaviors that can be observed and tested. Also, they suggest, to some extent, ways in which the ideas from Section I can be integrated into the teaching of mathematical content from each strand. It is our hope that both teachers and testing agencies will consider all of these statements when they design their tests. A careful study of these objectives will show that students are expected to master certain concepts, principles and skills. Middle school staffs might find it helpful to prepare a separate list of these to serve as a checklist for their students as they study the sequence of mathematics courses in their school. Also, educators will need to make decisions about which mathematical processes and content should be taught in the various middle school mathematics courses.

Problem Solving Skills and Strategies

A problem-solving approach, as noted in Section I, should permeate the instruction of all mathematics taught in the middle school. However, time should be set aside to teach problem-solving skills directly. In addition, a place should be found in the program for the inclusion of non-routine problems. During this time, students can apply sequences of problem-solving skills in planning their own solution strategies while working individually or in small groups. Their strategies also might include the use of the calculator and computer. Class time should be spent on comparing the problem-solving strategies used in the various solution attempts. Encourage students to keep written records of the solution paths (including blind alleys) they took, along with their personal or emotional reactions along the way.

At the end of the 8th grade, students should be able to:

- Pose and solve problems using skills such as guessing and checking, making a systematic list or table, looking for patterns, making a drawing or model, eliminating possible answers, solving a simpler problem.
- Give and write accounts of the solution strategies (both successful and unsuccessful) in the search for solutions to routine and non-routine problems.
- Use small-group problem solving and compare solution strategies with another group.
- Solve two- and three-step word problems, including those involving money, measurement, decimals, percent, commonly-used fractions and consumer applications.
- Sort and identify information necessary to solve word problems, and supply reasonable additional information if needed.
- Create word problems to match arithmetic equations involving whole numbers, commonly-used fractions, decimals and percent.
- Use an electronic calculator or devices for checking a guess or estimate.
- Write a calculator or computer program for a problem involving three or more operations, and solve.

Number and Numeration: Basic Concepts, Principles and Meanings

In a position paper entitled "A Proposed Set of Goals for High Schools As Students Prepare to Enter the 21st Century," Del Schalock writes:

> The mastery of a particular bit or area of knowledge at a level that makes it genuinely functional in one's life requires that it be understood thoroughly, that it be connected to related bodies of thought, and that it be integrated with other knowledge/attitudes/perceptions the student holds. Mastery of knowledge means more than recognition or recall on a test taken over a topic just studied.
>
> The mastery of a particular skill at a level that makes it genuinely functional in one's life requires that the skill can be used at a level of proficiency desired in a number of different contexts or situations, and over an extended period of time. Mastery of a skill means more than a one-time demonstration of the skill under simplified conditions.

Specifically applied to mathematics, this means that middle school students need to apply in a conscious way the basics of number and numeration if they are to be masterful users of the algorithms they are learning. Many of the basic principles implicit in the objectives which follow have special names such as distributive, associative, and commutative properties. A working knowledge of the principles, however, is more important than merely knowing their names.

At the end of the 8th grade, students should be able to:

- Write a number using the Egyptian, Mayan, or a numeration system of another culture.
- Compare our numeration system with an ancient system as to base, place value, and the existence of a symbol for zero.
- Write a number using a modern, non-decimal numeration system.
- Describe the usefulness of modern, non-decimal numeration systems in the structure of computers.
- Explain why rules need to be made concerning the order of operations in arithmetic expressions such as 13 + 7 x 5 - 1.
- Use the "order of operation" rules for calculating the answer for a string of successive operations.
- Write the "family" of facts associated with a given whole number fact: e.g., given 4 x 6 = 24, three other facts in the family are 6 x 4 = 24; 24 ÷ 4 = 6; 24 ÷ 6 = 4
- Explain why there are just three facts in the family associated with a given multiplication fact involving two zeros: e.g., given 7 x 0 = 0, two other facts in the family are 0 x 7 = 0 and 0 ÷ 7 = 0. Explain why the fourth expression 0 ÷ 0 = 7 does not make sense.
- Explain why dividing a natural number by zero causes difficulties.
- Give examples of subtraction and division exercises in which the answer depends upon the order of doing the exercise:
  e.g., 15 ÷ 3 = 5 but 3 ÷ 15 = 3/15 or 1/5
  18 - 10 - 3 = 5 if the subtractions are done from left to right, but equal 11 if 3 is first subtracted from 10.
- Explain how the properties of one and zero are helpful in learning the basic facts of arithmetic:
  e.g., multiplying any number by zero gives a product of zero.
  multiplying any number by one gives back that same number.
- Give examples of computations which force us to have answers other than whole numbers:
  e.g., 3 - 5 = -2 (-2 is not a whole number)
  15 ÷ 4 = 3 3/4 (3 3/4 is not a whole number)
- Explain why in 18 + (10 + 6) and 48 x (6 x 2) the parentheses can be safely ignored but not in 18 - (10 - 6) and 48 ÷ (6 ÷ 2).

- Explain how the order properties for adding and multiplication can be used in estimating answers and in making computations mentally, with a calculator and with paper and pencil:
  e.g., in $8 + 7 + 9 + 2 + 3$, first adding 8 and 2 then 7 and 3 is easier than adding in the designated order.

- Describe how a third number in a computation fact changes if the other two numbers are changed in a well-defined way:
  e.g., in $3 + 4 = 7$, if both 3 and 4 are doubled, the answer is also doubled. In $3 \times 4 = 12$, if both 3 and 4 are doubled, the answer is quadrupled.

- Describe how two of the three numbers in a computational fact can vary if the other number must always stay the same:
  e.g., in $20 \div 4 = 5$, the answer 5 remains the same as long as 20 and 4 are multiplied by the same number. In $20 - 4 = 16$, the answer remains the same as long as the same number is added or subtracted from 20 and 4.

- Express a whole number greater than 1 that is not prime as a product of primes.

- Estimate the value of a computation exercise and use a computer or calculator to calculate the answer and determine the error. For example: $9(98 + 5)/403$

- Sequence a set of numbers according to ascending or descending order (whole numbers, fractions, decimals, or integers).

- Write story problems for a
  - Division exercise using both "repeated subtraction" and "equal sharing" interpretations.
  - Subtraction exercise using a comparison interpretation:
    e.g., how much more money does Mary have than John?

- Make a drawing to illustrate statements such as

  $3 \div 4 = \frac{3}{4}$ or $3 \div 4 = 3(\frac{1}{4})$

  $3\frac{1}{2} \times 4\frac{1}{2} = 15\frac{3}{4}$

  $\frac{2}{3} \times \frac{1}{2} = \frac{2}{6}$ or $\frac{1}{3}$

- Give examples of multiplication exercises for which the product will be less than at least one of the factors:
  e.g., $12 \times .3 = 3.6$ $\quad$ $.4 \times .3 = .12$

- Give examples of division exercises for which the quotient will be more than at least one of the two given numbers:
  e.g., $12 \div \frac{1}{2} = 24$ $\quad$ $1.5 \div .3 = 5$

- Demonstrate why, for example, $\frac{2}{3}$ cannot be represented exactly by a decimal on a calculator.

- Use a calculator to change a fraction to a decimal rounded, for example, to the ten-thousandths place.

Appropriate Computational Skills

Mastery of computational skills is a necessary outcome of middle school mathematics. According to the National Council of Supervisors of Mathematics, "Position Paper on Basic Skills," computational skills need to be related to real-world situations and seen as a means of enhancing a person's ability to use mathematics in daily living. Computational skills include mental arithmetic, estimation, and calculator use as well as paper and pencil computation. Furthermore, instruction should include opportunities to select which mode of computation is more appropriate and whether or not answers are reasonable.

At the end of the 8th grade, students should be able to:

- Multiply and divide by 10, 100, and 1000 mentally and give answers as whole numbers or whole numbers with a remainder or as a decimal.

- Mentally add or subtract a power of ten from a large number.

- Determine whether a whole number is divisible by 2, 3, 4, 5, 9 or 10.

- Round numbers to the nearest designated unit. Truncate numbers.

- Determine how large or how small a rounded number could actually have been.

- Perform whole-number algorithms using:
  - Mental arithmetic
  - Estimation
  - A calculator or computer
  - Paper and pencil.

- Perform the four basic operations with "ruler" fractions and mixed numbers using:
  - Intuitive methods meaningful to students
  - Estimation procedures
  - Paper and pencil algorithms.

- Perform the four basic operations with decimals by using:
  - Mental arithmetic
  - Estimation
  - A calculator or computer
  - Paper and pencil.

- Demonstrate how the "putting together" interpretation of addition needs to be expanded to including adding with integers on the number line.

- Demonstrate how the repeated addition interpretation of multiplication needs to be expanded to include multiplying with fractions.

- Demonstrate how the "take away" interpretation of subtraction needs to be revised so as to include finding the difference between negative and positive temperature readings or differences on the number line.

- Demonstrate how the "repeated subtraction," "equal sharing," and ratio interpretations of division are needed in the search for applications for division of fractions.

- Select (or invent) story problems which match exercises or arithmetic equations involving whole numbers, decimals, ruler fractions, and mixed numbers and the four basic operations, and vice versa.

- Add and subtract integers using thermometers, above and below sea level, or other settings.

- Determine the time interval between two instances of time by referring to a calendar or clock.

- Determine the date or clock time of an interval backwards or forwards from a given point in time.

- Demonstrate how a calculator can be used on a series of operations such as 15 - 4 x 3 + 7 to get an answer of 10 rather than 40.

Two- and Three-Dimensional Geometry

We live on a sphere called Earth and work in a three-dimensional world. Citizens, consumers, and workers require some knowledge of geometry. Much more knowledge is needed in occupations such as plumbing, carpentry, forestry, interior decorating, architecture and engineering. Much of our information about our world is obtained from and recorded on two-dimensional drawings and pictures. The planning done to make most of our buildings and other structures is first done on two-dimensional charts such as blueprints. In addition then to acquiring knowledge of certain geometric concepts and properties and their applications, students need to develop their spatial and visualization skills.

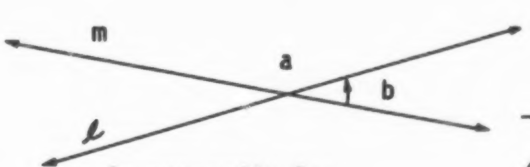
Geometric concepts should be integrated into many topics at each middle school level but, on occasion, should be treated as topics whose primary focus is some geometric topic. The instructional approach should emphasize exploration, informal reasoning, and the use of problem-solving skills. Much use should be made of tools which aid in geometric explorations, including the compass, protractor, the straightedge, grid paper, and the computer along with a programming language such as LOGO.

(Before determining the geometric content for their middle schools, mathematics teachers should study the recommendations made in "Oregon Mathematics Concept Paper No. 3: Geometry, K-12.)

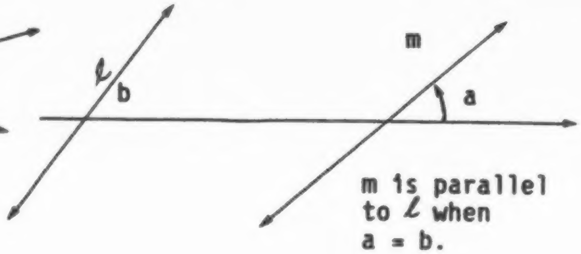
At the end of the 8th grade, students should be able to:

- Demonstrate with objects, drawings, or examples the ways lines and planes in space can be positioned with respect to each other: e.g., parallel planes and lines, skew lines, intersecting lines and planes.

- Illustrate angle and possible line relationships as a line rotates about a point on another line: e.g.,



m is perpendicular
to ℓ when a = b.
Also, a + b = 180°



m is parallel
to ℓ when
a = b.

- Give the common and distinguishing features of two geometric objects from drawings and 3-D models: e.g., a trapezoid and parallelogram; a cylinder and a cone.

- Classify polygons as to side or angle measurements (especially triangles and quadrilaterals).

- Make 2-D sketches of geometric solids such as cubes, pyramids, cones, spheres.

- Make drawings of 2-D figures which have point or line symmetry, including certain triangles, quadrilaterals, and closed curves.

- Select from a collection of 3-D models those which are symmetric with respect to a point, a line, or plane.

- Construct a stack of cubes given pictures of front, top, and side views, and vice versa.

- Show by using a drawing or model how a polygon with more than three sides can be made into a rigid figure by using braces to form triangular regions within the polygon.

- Explain the importance of using the triangle in construction and surveying.

- Use in a variety of situations involving triangles the:
  - Angle sum property
  - Side property
  - Conditions for similarity
  - Conditions for congruence.

- Inscribe a regular polygon within a circle using a compass, ruler and/or protractor.

- Determine the distance traversed by a wheel by counting the number of rotations and using the relationship between its circumference and diameter.

- Demonstrate how a circular region can be "cut up" to approximate the interior of a parallelogram and then relate this to a rule for finding the area of the region.

- Calculate the central angle that subtends a side of a regular inscribed polygon.

- Locate and give coordinates of points on maps, graph paper, globes.

- Plot triangles and other polygons on a grid when coordinates of vertices are given and vice versa.

- Plot (or graph) a triangle congruent to another triangle whose vertices have integral coefficients as coordinates.

- Use protractor, compass, ruler, and other instruments to construct a triangle similar or congruent to another triangle.

- Use drawing or cutouts to show how the area of a parallelogram can be found by finding the area of a rectangle; a triangle or trapezoid by finding the area of a rectangle or parallelogram.

- Build polyhedrons by using ready-made patterns (or nets).

- Draw several different patterns (or nets) which can be folded to form an open box with a rectangular base.

- Demonstrate and explain why a flat surface can be covered with congruent triangular tile but not by pentagonal tile.

Measurement

Measurement concepts surround us. We use an understanding of measures to quantify and interpret the world. Modern technology is totally dependent upon measurement. Hidden in most of humanity's spectacular accomplishments are innumerable measurements, each related to or dependent upon a myriad of other measurements.

Measurement can be taught more successfully if estimation is one kind of instructional activity. To teach measurement we should be concerned with more than a system of measures. We must teach a "doing" kind of mathematics. Activity gives meaning to measuring skills, makes the resultant learning personally satisfying to the pupil and begins the development of a process that will be used throughout life. (Based upon comments from the NCTM 1976 Yearbook, Measurement In School Mathematics.)

(Before determining the measurement content, middle school mathematics teachers should study the recommendations made in "Oregon Mathematics Concept Paper No. 3: Geometry, K-12.)

At the end of the 8th grade, students should be able to:

- Explain why there is a need for standard units of measurement.
- Give examples of situations in which nonstandard units can be used as efficiently as standard units.
- Illustrate by some means the size of basic English and metric units without the presence of measuring devices.
- Estimate distances and other measurements.
- Determine the most appropriate unit for a measurement task.
- Measure directly distances to the nearest and smallest unit indicated on the measuring scale and indicate in some way the precision of the measurement (both English and metric).
- Indicate in some way that measurements are approximations.
- Measure angles to the nearest degree using a protractor.
- Measure by using a transparent grid the area of a polygon or some 2-D region with curves as boundaries.
- Measure by using unit cubes (or some other unit) the volume of 3-D objects.
- Determine the number of significant digits in a measurement taken or recorded to the nearest unit.
- Find measurements including inaccessible distances indirectly using formulas for distances, area, and volume, and record the measurements using the appropriate unit of measure.
- Show in some way that calculations with measurements often indicate a misleading degree of accuracy or precision:
  e.g., the length and width to the nearest foot of a rectangular panel are 6 ft. and 4 ft. The actual area in all probability is not 24 sq. ft. but somewhere between $19\frac{1}{4}$ sq. ft. and $29\frac{1}{4}$ sq. ft.

  $$5\frac{1}{2} \times 3\frac{1}{2} = 19\frac{1}{4} \qquad 6\frac{1}{2} \times 4\frac{1}{2} = 29\frac{1}{4}$$

- Give examples of the importance of using congruence and precision concepts in our society: e.g., replacement parts for our appliances must fit if they are to work properly.
- Illustrate how the perimeter, area, and volume of geometric shapes changes as the dimensions are doubled or tripled (attention given primarily to rectangles and prisms).
- Make scale drawings.

- Determine actual distances (including highway distances) from scale drawings: e.g., blueprints, maps, globes.

Probability and Statistics

We live in a society in which we are confronted daily with quantitative information or data. Statistics is the science or study of data. Quantitative literacy is a requirement for all educated individuals who want to be informed citizens and hold jobs in technical businesses and industries. Throughout life, decisions about health, citizenship, parenthood, employment, financial concerns, and sports are based upon quantitative information. Statistics is most often concerned with using information in the face of uncertainty. Probability gives us a way to measure uncertainty.

In addition, studying statistics and probability can help in the development of students' critical thinking skills. In carrying out experiments, students develop ways to cope with uncertainty as they are searching for truth in a situation and learning to report it faithfully. Approaching situations statistically can help make students face up to their own prejudices, think more consistently about arguments, and justify their thinking with numerical information. (Adapted from a paper written by Marilyn Burns.)

(Before determining the probability and statistics content, middle school mathematics teachers should study the recommendations made in "Oregon Mathematics Concept Paper No. 4: Statistics and Probability, K-12.)

At the end of the 8th grade, students should be able to:

- Read, make interpretations and predictions from graphs, tables, and charts.
- Use data-gathering procedures which will help in answering questions of interest: e.g., conducting polls, sampling schemes, using tables from almanacs.
- Organize data to support conclusions or help answer questions.
- Discuss methods of presenting data which will be most effective in supporting a conclusion.
- Search for data which might contradict certain stated conclusions.
- Identify misleading or incorrect methods of displaying data.
- Calculate mean, median, and mode for data taken primarily from situations of interest. (Use a calculator when it is quicker to do so.)
- Determine and then justify which measure of central tendency (mean, median, or mode) is the best representation of the data.
- Give examples of how measures of central tendency can be a distortion of the truth.

- Give examples of different ways the term "average" is used in newspaper articles.

- Estimate the chances of an event occurring and then conduct an experiment to determine the accuracy of the prediction.

- Use a computer to generate random circumstances to supply data for probability experiments: e.g., 1000 coin tosses or 5000 picks of playing cards.

- Use a tree diagram (or some other systematic listing) for determining the number of possibilities that could occur: e.g., the different sums of the numbers showing on the "upside" of two number cubes.

Patterns and Relationships

Variation

Formulas, tables, and graphs often have a close relationship to the "real world" and frequently can be used in making useful predictions. Topics in this unit can be used to show how one thing changes as another changes. Students should realize from such a unit that formulas have a limited range of applicability and though useful, seldom exactly describe data collected from the real world. They should realize that caution should be exercised when making predictions from tables, graphs, and formulas. Students should have opportunities to organize and make predictions based upon data they collect.

At the end of the 8th grade, students should be able to:

- Evaluate formulas which are likely to have some meaning to the pupils: e.g., $A = lw$, $D = rt$, $A = \pi r^2$, $V = lwh$.

- Make tables of values for a formula.

- Write a formula for a table of values.

- Create a story problem for a formula or an equation.

- Make a graph with its formula.

- Describe, when given a table, graph, or formula of a two-variable situation, how one variable changes as the other changes.

- Predict whether or not the graph for a table suggests a straight or curved line.

- Describe situations in which formulas or tables do not always accurately describe things as they really are: e.g., age/hours of sleep formula; weight/height formulas for young people.

Ratio, Proportion, and Percent

Few topics in mathematics have more applications to the lives of citizens than ratio, proportion and percent. Our students are frequently exposed to these concepts in science, home economics, and industrial art classes. The language is used in social studies classes and athletes are interested in improving their averages. Musicians and artists are concerned about ratios and the right proportions.

Percent is an excellent topic for pulling together concepts of fractions, decimals, and ratio. Also, "real life" examples of percent lends itself to estimation and approximation.

Even though ratio, proportion, and percent are pervasive in our culture, our students and the public in general have many difficulties with these concepts. For this reason, we as teachers need to be cautious in our approaches to teaching these concepts. It is more difficult for students than we think. We need to be certain that our teaching approaches take into account the levels of understanding of our students.

At the end of the 8th grade, students should be able to:

- Show how a portion of a pie chart labeled with its appropriate fraction can be interpreted as a ratio.
- Show how a fraction represented by a point on a number line can be interpreted as a ratio.
- Give examples of measurement situations in which two variables always have:
  - A constant difference: e.g., ages of two persons as they grow older.
  - A constant quotient (or ratio): e.g., the total cost and the number of items purchased.
- Find the missing number in a proportion exercise.
- Select from several sets of four numbers those sets which can be used to form a proportion.
- Given a proportion exercise with a missing term, invent or create a problem that can be solved by using the proportion, and conversely.
- Given a proportion, rearrange the four numbers to form at least two other proportions.
- Identify from a page of miscellaneous review story problems those which can be solved by using a proportion.
- Use proportions to solve the lengths or sides involving similar triangles.
- Answer questions concerning percent using pupils' intuitive understandings of percent, 100-grid, circle diagrams, and/or number lines.

- Use the above models to show how percent can be expressed as a fraction or decimal, or conversely.
- Use the "for every hundred" interpretation for solving percent problems.
- Use the 1 percent method for solving percent problems.
- Solve percent problems by translating to a proportion with 100 as one of the four terms.
- Given a proportion exercise with 100 as one of the terms, invent a percent story problem that matches a proportion.
- Solve simple percent problems mentally.
- Solve simple interest, discount, tax, and percent of increase/decrease problems.
- Estimate the solution to percent problems.
- Solve certain percent problems by first converting the percent to a fraction.
- Solve percent problems (including change a fraction to a percent) using a calculator.

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SECTION III - CONCERNS, ISSUES, AND POINTS OF VIEW

As a community and its educational leaders make plans to improve their middle school mathematics program, they need to consider the nature of instruction and the content they wish to have taught. Certainly they should consider the needs of the local community, the state, and the nation at large. In this age of technology, nearly everyone believes that changes need to be made. Generally, we must place emphasis on those things which people can do better than machines. The OMEC has made its views known on these matters in the previous sections. What follows in this section are our views of some other concerns.

1. Personal Finance/Consumer Topics

What personal finance/consumer topics shall be included in the middle school mathematics program?

Each year a certain percentage of students will drop out of high school. For many of these students the 8th grade course will be their final experience in mathematics. Thus, at least an introduction to some important consumer topics is essential for students at the 8th grade level.

It is quite possible, also, that many students who move into a crowded college preparatory track in high school will not experience certain important personal finance/consumer topics unless at least some introduction is given in 8th grade.

Personal finance/consumer topics which all middle school students should be exposed to include a basic introduction to:

- Unit pricing.
- Budget preparation.
- Installment buying.
- Single and compound interest.
- Checkbook balancing.
- Taxes.
- Payroll math.

In developing units in personal finance/consumer topics, teachers need to include many opportunities for students to experience:

- Estimation and mental computation.
- The use of the calculator.
- Interpreting charts, graphs, and formulas.
- Application of ratio, proportion, and percent.

2. Self-Paced Instruction

To what extent should individual self-paced instruction be used as a method for allowing for individual differences among students?

There is no question that individualization is necessary if good instruction is to occur. It is quite obvious that some students need more drill and practice than others. Some need to have more manipulative experiences. Some require more individual coaching sessions from the teacher or from a classmate. And some desperately need further extensions to challenge problem investigations.

A good mathematics program will allow for all of these individual needs. Nevertheless, the usual interpretation of "individualization" does not provide for this type of individualization to take place. Instead, students often work through text material at their own rate which too often results in a limited understanding of some very important mathematical concepts.

In an individual self-paced program such as this, the students are very often working on completely different problems and from different levels of the curriculum. The only "help" the teacher can give usually amounts to a rushed "show and tell" with the teacher talking and the student more or less listening. The teacher would like to diagnose the student's difficulties and give the necessary recommendation, but there is no time--another student is waiting. This process is inefficient since the same "help" must be provided on a one-to-one basis again and again within a short span of time.

Research has shown that students in an individual self-paced program such as the one described here generally do not perform as well as those students learning under the normal style of instruction.

Indeed, students do learn mathematics at different rates. But instead of accelerating their learning in a linear direction, a more reasonable solution is to provide enrichment activities in which students have many opportunities to solve problems and apply the mathematics they have learned.

3. Vertical Acceleration or Enrichment

To what extent and how should vertical acceleration be used for high-achieving and mathematically-talented students? We recommend the position taken by the National Council of Teachers of Mathematics, in April of 1983, in which they state:

> That vertical acceleration be considered only for a limited number of highly talented and mathematically creative students whose interest and attitudes clearly indicate that they have the ability and perseverance to complete a carefully designed sequential curriculum. For all but this select group, a strong expanded program emphasizing mathematics enrichment is preferable.
>
> Official NCTM position, April 1983

In general, this task force agrees with this position. The above statement and a discussion on the acceleration/enrichment question are included in an OMEC unpublished paper available from Don Fineran, Oregon Department of Education.

This middle school mathematics program as described in this OMEC document is a strong expanded program emphasizing enrichment possibilities in many important areas. This program advocates the inclusion for each grade level course mathematics topics which can be taught by informal approaches that capitalize upon understandings that most students already have. The mathematics topics for these middle school courses are from many branches of mathematics, rather than from just arithmetic, and most are open-ended. Also, an emphasis is placed upon mental arithmetic, estimation and approximation, and the use of calculator and computer. In addition, the instructional approach suggested in this OMEC document encourages students to think mathematically, with explicit attention given to problem solving processes and the search for problems by students themselves.

Such a middle school mathematics program is appropriate for all students and can be achieved by the end of the 8th grade by a large majority of students in classes with their grade-level peers. The above-average student will need a planned program of enrichment which would evolve from the open-ended topics included in the recommended program.

While enrichment is suggested as the main vehicle for allowing for individual differences, we do recommend that certain capable, high mathematics achievers be allowed to elect a course of study which accomplishes in one year the mathematics objectives suggested for two years. Such a move, however, should be delayed as long as possible. In most middle schools such a "two-year-in-one" course would not be advisable until 7th grade. A separate class might be scheduled for such a course, or students in the 7th grade class might be grouped for such a two-in-one program. Questions such as the following should be satisfactorily answered before a student is placed in this course.

- Is the student able to work effectively individually and in small groups?
- Does the student indicate that he/she can read with comprehension?
- Does the student enjoy working challenging problems?
- Has the student demonstrated a willingness to do homework?
- Does the student's past high achievement extend beyond high arithmetic computation scores?

Students who successfully complete this two-in-one course should be allowed to enroll as 8th graders in the usual algebra course or some suitable equivalent (see "Oregon Mathematics Concept Paper No. 5: An Integrated Mathematics Curriculum for High School").

Another form of acceleration for the middle school is to allow students to skip a grade-level course beyond his or her current grade level. Any such skipping should be limited to those highly-talented and highly-creative students (usually not more than one or two percent of a given grade level) whose attitude and interest clearly indicates they have the ability and perseverance to complete a highly-sequential program. For the most part, such a program would need to be planned with the individual student in mind and monitored by a person with a specialized background in mathematics.

4. Low Achieving Mathematics Students

Should all low-achieving mathematics students be kept in mathematics classes with their "age-level" peers? If not, what mathematics should these students study and in what class should they be placed?

These questions are not easy to answer. Whatever answer one chooses to accept should be flexible in its application.

Low achievement in mathematics is not always the result of low innate ability. Irregular attendance might be a cause as might be the lack of continuity of instruction due to the mobility of the family from one school district to another, varying rates of pupil cognitive and emotional development, and teaching approaches inconsistent with the pupils' learning styles.

Two widely-held and somewhat contradictory views need to be kept in mind as school staffs plan their programs for low-achieving students. Below-average students should be kept with their age-level peers as long as possible. On the other hand, these students should be studying mathematics in classes where achievement is possible. Unsuccessful experiences have a cumulative effect, with each failure making it more difficult for students to develop confidence in their ability to do mathematics.

Much can be done for low-achieving students in the regular grade-level classes if:

- Instruction takes into account the intuitive understandings students have gained from their previous experiences both in and out of school.
- Appropriate concrete materials are used in the development of concepts.
- Use is made of the calculator.
- The grade-level program includes content from various strands of mathematics rather than only from the computational aspects of arithmetic.
- Many of the topics studied do not require the mastery of a hierarchy of skills as prerequisites.
- Classroom management practices include the use of cooperative learning schemes such as groups-of-four or partner-learning units.
- Individualized activities or assignments are made which are related to the topic being studied by the class at large.

There is a time, however, when certain low achievers should be provided with instruction more commensurate with their demonstrated achievement than can be provided in a class with their grade-level peers. If pupils are a year or more behind the expected mathematical achievement level for entering middle schoolers, they can expect few, if any, positive experiences in the first "regular" middle school course. Some of these pupils (usually the least successful) may be placed in a Chapter I resource room where appropriate developmental instruction can be provided individually or in small groups. The larger group of remaining pupils would profit from a special course where pupils are provided with instruction which would increase their chances of succeeding in the first "regular" course a year later. The content would be selected from a variety of elementary school mathematics topics. These classes should have no more than fifteen pupils and the instructional approach should use the strength of the students to help overcome their weaknesses. Teachers for such a course would need to be able to diagnose difficulties pupils have and to select* and design appropriate curriculum materials for their use.

5. Algebra in the Middle School

How early and in what form should algebra be taught in the middle school?

Selected algebraic content should be integrated into the content of all middle school mathematics courses. The emphases should be on variation (patterns, graphs, tables, and formulas), on properties and relationships involving operations with numbers, and on the interrelationship between arithmetic, statistics, geometry and probability.

Nearly all current first-year algebra texts are highly oriented toward technical skills with few applications and problem solving opportunities. Also, research evidence seems to suggest that most of the content in these texts is comprehensible only to those students in the formal operation stage of cognitive development--a stage beyond nearly all middle school students. For these reasons, we recommend that students defer taking this type of algebra course until after they have demonstrated a mastery of the content strands suggested in Section II of this report.

6. Effective Instruction

What relevance do the research findings on effective instruction have on teaching middle school mathematics?

Recent research on instruction shows that we can improve teaching and make a difference in student achievement, attitude, and conduct. Much of this information comes to us from watching what effective teachers are already doing. These teachers are more active, ask more questions, give more opportunity for response, and spend more time in class discussion than do their less effective counterparts. They also give encouragement, have high expectations, model problem solving behavior, use many examples and counter-examples and are clear in their explanations. Their teaching includes demonstration, practice, feedback, application, and follow-up assistance as part of the plan.

In middle school classrooms we will need to incorporate active teaching strategies. We will need to keep students involved in their own learning by giving them ample opportunities to participate in class with frequent review, controlled practice of new concepts, and opportunities to discover new methods of solution by carefully planning activities. Variety in techniques and materials will be used within class periods and from day to day.

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*See list of resources, pp. 78-81.

Students must see a reason for studying each topic, and it must make sense to them. Thus, we must seek relevance through application and develop critical thinking skills through problem solving opportunities. By incorporating these strategies into our teaching in the middle school mathematics program, we can positively effect change in achievement, attitude, and behavior.

7. Inservice and Staff Development

What inservice and staff development is needed to implement this recommended program?

> Substantial, continuous staff development is essential to the improvement of schooling and, equally important, to the development of capability for the continuous renewal of education.
>
> Bruce Joyce

In order to implement the recommendations in the middle school curriculum, teachers will need the opportunity to meet, learn, discuss, find or develop appropriate materials, and implement these topics. This staff development must be an integral part of the mathematics teacher's responsibility, easily accessible to the teachers involved, and must be responsive to change. These efforts should be needs-based and collaborative in nature.

Specific activities/workshops/inservices that middle school mathematics teachers should consider for implementation of the curriculum recommended in this paper would include:

1. Developing competency in problem solving. Teachers should know what competencies problem solving includes, should be good problem solvers themselves, and should have or develop materials for implementing problem solving strategies into their regular teaching practice.

2. Using concrete materials. Middle school mathematics teachers should know what materials are available or can be made and where their application can help students to bridge the gap from the world they see to the world they deal with symbolically. Teachers should make a collection of these manipulatives for their classrooms.

3. Learning and practicing effective teaching strategies. First, teachers must learn what teaching strategies have proven effective. Second, they must practice these strategies. A collegial supervision effort within districts might be a possible way to implement new techniques. These efforts would be for the improvement of instruction and not end-of-the-year evaluation. Release time should be provided to implement these techniques.

4. Developing and/or collecting activities for mental arithmetic and estimation activities and drills.

5. Maintaining a collection of application activities for all topics covered in the mathematics class.

6. Becoming conversant with classroom management techniques that allow for the use of laboratory settings, peer teaching, calculators, and computers.

7. Developing calculator and computer literacy by teachers when they are able to implement the use of these tools in their classrooms. Materials and activities must be developed/collected to be available for use when planning units.

8. Assembling collections of media aids and strategies. Mathematics classrooms should be interesting places to be. Bulletin boards should be interesting. They may show where mathematics is necessary in careers and daily life, history of mathematics, mathematics in art and nature, and problem solving activities. Board space should be used, overhead projectors available for use for prepared materials, and posters should be evident.

This list is not meant to be conclusive, but a source of ideas for staff development activities.

8. Role of the Textbook

What is the role of the textbook in the recommended middle school program?

> The textbook is a tool for teaching, a basic resource but one that must be used thoughtfully and with discretion. At worst, it can serve either as a dust collector and exercise book or on the other hand as the course--unmodified and often quite inappropriate. At best, it is an integral part of a total program, drawn upon when its use is most appropriate, but not called upon to perform tasks for which it was not written. The text provides foundations on which student and teacher build.
>
> Johnson and Rising. Guidelines for Teaching Mathematics, Second Edition, 1972, p. 378.

The middle school program, as recommended in this report, encompasses a broad range of mathematical topics. It is unlikely that any one textbook will be suitable for all topics. Thus, a good set of supplementary materials is essential (see section on Teacher and Pupil Materials, p. 79).

Problem solving is the general theme throughout this entire middle school mathematics program. Therefore, when selecting an appropriate textbook, this problem solving focus should be considered as the most important feature. In addition, a textbook which presents an informal activity-oriented approach throughout is highly recommended and would fit closely with the spirit pervading this report.

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An appropriate teachers' guide should include classroom management recommendations, suggestions pertaining to teaching strategies and techniques, and most important, suggestions for supplementary materials including appropriate laboratory "hands-on" activities.

A position paper published by the National Council of Teachers of Mathematics addresses the problem of "Selection and Implementation of Instructional Materials." This paper highlights the importance of evaluating instructional materials according to:

- Content coverage and development of each major curricular strand
- Development of the full range of cognitive abilities, especially problem solving and analysis
- Correlation of the content with local and state scope-and-sequence guidelines
- Correlation of the content and instructional approach with research on mathematics teaching and learning.

9. Teacher and Pupil Materials

What materials are available to help implement this program?

The district-adopted textbook is the basic resource for most middle school mathematics teachers. The teacher's manual provides an organizational structure and reference for the teacher. The pupil book serves as a basic resource for the student. Although the textbook is an integral part of the mathematics program, it seldom contains the resources needed to provide a complete program of instruction.

Certain content strands and instructional themes detailed in this document are more likely to require support materials than are others. The textbook, for example, is likely to provide adequate drill and practice with paper and pencil computation, but unlikely to have sufficient mental arithmetic exercises. Likewise, most operations with whole numbers, fractions, decimals, and percent are apt to be "covered" by the textbook, but appropriate resources for helping pupils understand and retain the underlying concepts may not be provided. The textbook may contain adequate word and story problems, but provide minimum instruction in the skills needed for non-routine problem solving.

The list that follows suggests a variety of resources that middle school teachers will find useful in augmenting the textbook and providing a more complete program of instruction. Following each entry is a coded indication of the major content strand or instructional theme provided by that resource. The code is as follows:

Content strands

CS 1. Problem Solving Skills and Strategies
CS 2. Number and Numeration: Basic Concepts, Principles and Meanings
CS 3. Appropriate Computational Skills
CS 4. Two- and Three-Dimensional Geometry

CS 5. Measurement
CS 6. Probability and Statistics
CS 7. Patterns and Relationships
  Variation
  Ratio, Proportion and Percent

Instructional themes

IT 1. Problem Solving
IT 2. Applying Mathematics
IT 3. Mathematical Thinking
IT 4. Bridging the Gap Between the Concrete and the Abstract
IT 5. Developing Concepts
IT 6. Developing the Ability to Make Estimations
IT 7. Relational Thinking
IT 8. Using Computer and Calculators
IT 9. Reading, Writing, and Talking about Mathematics

Equals Project. *Cooperative Logic Problems*. Lawrence Hall of Science, Berkeley, CA. (CS 1, IT 1, IT 2, IT 3, IT 9)

Lane County Mathematics Project. *Problem Solving in Mathematics, Grades 4-9*. Dale Seymour Publications, Palo Alto, CA. (CS 1, CS 2, CS 3, CS 4, CS 5, CS 6, CS 7, IT 1, IT 2, IT 3, IT 4, IT 5, IT 6, IT 7, IT 8, IT 9)

Laycock, Mary and Margaret Smart. *Solid Sense In Mathematics*. Activity Resources Company, Inc., Hayward, CA. (CS 1, CS 2, CS 3, CS 4, CS 5, CS 7, IT 3, IT 4, IT 5, IT 9)

Mathematics Resource Project. *Geometry and Visualization*. Creative Publications, Palo Alto, CA. (CS 1, CS 4, CS 5, IT 1, IT 2, IT 3, IT 4, IT 5, IT 6, IT 7, IT 8, IT 9)

Mathematics Resource Project. *Mathematics In Science and Society*. Creative Publications, Palo Alto, CA. (CS 1, CS 3, CS 4, CS 5, CS 6, CS 7, IT 1, IT 2, IT 4, IT 6, IT 8, IT 9)

Mathematics Resource Project. *Number Sense and Arithmetic Skills*. Creative Publications, Palo Alto, CA. (CS 1, CS 2, CS 3, IT 1, IT 2, IT 3, IT 4, IT 5, IT 6, IT 8, IT 9)

Mathematics Resource Project. *Statistical and Informational Organization*. Creative Publications, Palo Alto, CA. (CS 1, CS 3, CS 6, CS 7, IT 1, IT 2, IT 4, IT 5, IT 6, IT 8, IT 9)

Mathematics Resource Project. *Ratio, Proportion and Scaling*. Creative Publications, Palo Alto, CA. (CS 1, CS 3, CS 4, CS 7, IT 1, IT 2, IT 3, IT 4, IT 5, IT 6, IT 7, IT 8, IT 9)

McFadden, Scott, Keith Anderson, and Oscar Schaaf. *Mathlab Junior High*. Action Math Associates, Inc., Eugene, OR. (CS 1, CS 2, CS 3, CS 4, CS 5, CS 6, IT 1, IT 2, IT 4, IT 5, IT 8, IT 9)

National Council of Teachers of Mathematics. The Arithmetic Teacher. Reston, VA. (All content strands and instructional themes)

National Council of Teachers of Mathematics. The Mathematics Teacher. Reston, VA. (All content strands and instructional themes)

Oregon Council of Teachers of Mathematics. The Oregon Mathematics Teacher. Portland, OR. (All content strands and instructional themes)

Reyes, Robert and Barbara Reyes. Guess: Guide to Using Estimation Skills and Strategies, Boxes 1 and 2. (CS 1, CS 2, CS 3, CS 5, CS 6, IT 1, IT 2, IT 3, IT 6)

10. Evaluation and Standardized Tests

How can mathematics achievement as outlined in this program be evaluated? To what extent can standardized achievement tests be used? Should calculators be used by students as they are being tested?

Educators and the general public alike are interested in how well our young people are doing in mathematics in our schools. The reasons for this interest are not always clear. Some might feel that achievement data help:

1. Citizens determine how effective schools are.
2. School officials or other citizens determine how their local school ranks with other schools according to objectives determined by some national testing agency.
3. Students decide which vocation they might pursue.
4. Counselors advise students on the next mathematics they might take.
5. School personnel determine how their mathematics program might be improved.
6. Mathematics teachers diagnose difficulties, determine what understandings students have, and plan instruction accordingly.
7. Parents determine the progress their children are making in their school work.

Of course, nearly all these reasons are sound and all must be respected. Educators need to search for and provide the type of information which will satisfy the concerns and help answer the questions these interested parties might have. It is important to realize that the information needed in each case will take a different form, depending upon the individual's reason for making the assessment.

Other important variables to keep in mind in evaluating a mathematics program are the goals and objectives of instruction. These should be stated clearly so students, teachers, and parents understand them. We tried to do this for each mathematical strand discussed in Section II of this report. Furthermore, information needs to be collected which can be used to assess the progress students are making toward achieving all the objectives. The mathematics program in this report advocates in addition to the acquisition of certain concepts, principles and skills, such process goals as mathematical thinking, problem solving, and applying mathematics. Teachers can make reasonably accurate judgments on the achievement of all these process goals and objectives by listening to students discuss their thinking strategies, watching them work individually and in small groups, reading what they write while doing and writing about mathematics, examining tests and quizzes they design, and studying an item analysis of the results of scores on carefully selected standardized achievement tests.

Reputable testing agencies take care in designing standardized mathematics tests with much attention given to validity, objectivity, and reliability of the items. The test manual specifies the conditions under which the test was constructed, the objective for each item is specified, the "p" values (the percent of students who answered the item correctly) for each test item, and the percentile rankings of all possible total scores. Nearly all tests use a multiple choice format. The correct answer (or the product of the student's thinking) is evaluated rather than the process used to obtain the answer.

Some testing companies have two types of mathematics tests available for a given level. One is an abbreviated form which can be taken during a single class period and includes but one item for each specified objective. The other type may take as many as three class periods with at least three items for each objective. Most school districts, because of time constraints, use the shorter test. The results from either test might provide some help in assessing certain product objectives, but only the longer test will be of value in helping teachers diagnose students' difficulties, plan instruction, and place students in appropriate classes.

National standardized tests are inadequate instruments for assessing the effectiveness of mathematics programs such as the one advocated in this OMEC document. Some, but not all of its goals and objectives are given limited attention; others such as paper and pencil computation are over-emphasized at the expense of other problem solving skills such as mental computation, effective use of the calculator, estimation and measurement. Furthermore, the multiple choice format of these tests does not lend itself to an adequate evaluation of the highly important process goals, namely mathematical thinking, problem solving, and many of the application objectives. An evaluation of these process goals requires careful observation of a student by a teacher sensitive to the nature of these goals.

Should standardized tests be used in evaluating this OMEC program? Our answer is yes with many reservations. The test results should be carefully analyzed and interpreted, keeping in mind the goals and objectives of this proposed program and the reasons for giving the test. Much attention should be given to the test's "p" values for the relevant items, with little or no serious attention given to the percentile rankings of individual student's or schools' total test scores.

Especially on those tests which have more than one item for each objective, "p" values provide some information on places where the mathematics program might be improved and where a student's mastery of an objective is apt to be weak or strong. Percentile scores--because of reasons stated above--would be misleading measurements of the effectiveness of this proposed program.

Should calculators be used by students as they are being tested? The answer is yes. Skillful use of a calculator is an important goal of this proposed program. On a standardized test, calculator use should be allowed on all but a few items.

11. Cooperative Learning

How and to what extent should cooperative learning be used in teaching the mathematics recommended in this program?

> Without the cooperation of its members society cannot survive, and the society of man has survived because the cooperativeness of its members made survival possible--it was not an advantageous individual here and there who did so, but the group. In human societies the individuals who are most likely to survive are those who are best enabled to do so by their group.
>
> Ashley Montagu, 1965

There are three basic ways pupils can interact with each other as they learn. They can compete to see who is "best;" they can work individually on their own toward a goal; or they can work cooperatively with a vested interest in each other's learning as well as their own.

In a cooperative working environment, pupils are arranged in small groups where they work cooperatively toward the solution of a task. This type of organization is particularly well suited for problem-solving activities because it invites the active participation of each pupil and provides pupils the opportunity to offer ideas and receive immediate feedback.

Cooperative learning improves how students feel about themselves, school, and others. There also is evidence that more pupils achieve at higher levels in cooperative learning situations and that they retain the information longer. Cooperative learning has also been shown to facilitate:

- Mastery of concepts and principles.
- Application of information to other settings.
- Problem-solving skills.
- Creativity.
- Verbal skills.
- Ability to take another person's perspective.

12. Use of the Computer

How should the computer be used in middle school mathematics programs? In what ways can the computer be used to enhance the implementation of this program?

The computer should be used in the middle school mathematics curriculum when it serves the goals and objectives described in this paper. Technology provides students with a wider variety of experiences and learning modalities. In addition, less time is needed to cover calculation skills and algorithms and more time spent on developing concepts, generalizations, models, and problem-solving skills.

Computer coursework can be used in one or more of the areas described below:

1. Problem Solving Activities--The computer is a problem solving tool. It can be used to collect and analyze data, test hypotheses, develop generalizations, and check work. Additionally, some software addresses specific problem solving strategies.

2. Simulating Real-World Situations--Simulations give students ample opportunity to practice computational skills, make mental estimations, develop problem solving strategies, and improve decision-making skills. This medium provides a bridge between abstract mathematical concepts and concrete problems.

3. Drill and Practice--If careful concept development has already taken place, using the computer for drill and practice provides the student with immediate, nonjudgmental feedback. Some programs keep records of student progress and provide review of basic principles when appropriate.

4. Programming--Simple programming skills are useful for exploring mathematics and using the computer as a sophisticated calculator. By empowering students with a minimal amount of programming experience, students can gain control over their own learning--writing programs that explore number relationships, developing algorithms, generating random or patterned numbers, and gaining experience in dealing with variables. The goal is to enhance mathematical learning, not teach complex programming precepts. It also allows ample opportunity for individual differences.

5. Tutorial--Computers can be used to present, review, or reinforce material in an interactive manner. It is useful in areas for individual enrichment or improvement.

6. Graphing Tools--Computer graphics support and underscore geometric concepts, spatial visualizations, transformations, relationships, and a multitude of graphing environments.

7. Spreadsheets--This organizational tool provides a format for representing and storing data. In the process of solving problems formulas are embedded, relationships analyzed, complex and multiple calculations made, and hypotheses tested.

8. Teacher Utilities--Word processors, graphic design tools, worksheet generators, gradebook and data base managers are a few of the applications teachers can use to save time, clarify written work and create new materials.

9. Exploratory Software--These pieces provide exploratory micro-worlds. Within these worlds students can pose and test hypotheses, discover relationships, and explore the creative process of developing mathematics.

Computers will never be a substitute for effective teaching, but they can be used to enhance the mathematics curriculum and instruction.

13. Middle School Mathematics Courses

What mathematics courses should be included in a middle school program?

The answers to questions 3, 4, and 5 imply that in addition to a Chapter I resource room (RR) there should be three regular middle school mathematics (MSM) and three other courses for low- and high-achieving students--call these respectively MSM-1, MSM-2, MSM-3, and MATH A, MATH B, MATH C. In general, the students in these courses will be from different grade levels. Some sequences of courses a student might take are shown below:

Grade	I	II	III	IV
6	MSM - 1	MSM - 1	MATH A	RR
7	MSM - 2	MATH B	MSM - 1	MATH A
8	MSM - 3	MATH C	MSM - 2	MSM - 1

MATH A - Content selected from a variety of elementary school mathematics topics (see p. 75)

MATH B - A two-in-one course for high achievers (see pp. 73 and 74)

MATH C - The first regular high school mathematics course, probably algebra (see pp. 74 and 76)

The placement of students in these courses should be the combined responsibility of teachers and counselors from both the feeder elementary schools and the middle school.

A MATHEMATICS CURRICULUM FOR OREGON MIDDLE SCHOOLS

BIBLIOGRAPHY

1. NCTM. The Place of Mathematics in Secondary Schools, 15th Yearbook, 1940.

The mathematics content of seventh and eighth grades should include topics in:

- Arithmetic
- Informal geometry
- Graphical representation
- Algebra
- Trigonometry
- Modes of thinking, habits and attitudes
- Historical backgrounds
- Correlation with life situations and other school activities

2. Oregon Department of Education, Mathematics in Oregon Secondary Schools, 1960.

> The mathematics content for the various courses should be selected from the point of view of a correlated approach. Such an approach would make it easier for students to see mathematics as an interrelated system of ideas. Arithmetic, algebra, geometry and trigonometry should be a part of the instruction at many levels rather than one level only.

3. Oregon Department of Education, MATH in Oregon Schools, 1977.

The document emphasizes a philosophy of mathematics with a focus on problem solving and advocates the following strands to serve as a common base in planning and developing several statewide efforts relating to math education.

- Numeration
- Communication through symbols and models
- Problem solving
- Computation
- Applications
- Geometry
- Measurement
- Estimation and approximation
- Probability and statistics

4. National Council of Supervisors of Mathematics. Position Paper on Basic Mathematics Skills, January 1977.

The NCSM views basic mathematical skills as falling under the following ten vital areas:

a. Problem solving
b. Applying mathematics to everyday situations
c. Alertness to the reasonableness of results

d. Estimation and approximation
e. Appropriate computational skills (mental, written, electronic)
f. Geometry
g. Measurement
h. Reading, interpreting and constructing tables, charts and graphs
i. Using mathematics to predict (probability)

Oregon Mathematics Concept Paper No. 3

GEOMETRY, K-12

PREFACE

The Oregon Mathematics Education Council (OMEC) launched a mathematics curriculum project in May 1984. Nearly 100 enthusiastic, dedicated volunteers, a majority of whom are classroom teachers, worked on the task forces of the project. These groups worked to develop a set of concept papers relating to major issues of reform in mathematics education. Their ideas are shared now as a set of The Oregon Mathematics Concept Papers. They are available as a complete set or as separate papers. You are encouraged to read those papers of special interest and to discuss the ideas with colleagues. Duplicating the papers for such noncommercial purposes is encouraged.

OMEC is a widely representative advisory group, representing teachers at all school levels, counselors, administrators, school boards, business, labor, industry, and parents. The Oregon Mathematics Concept Papers are being disseminated by the Oregon Mathematics Project, working with a number of mathematics organizations, and many individuals. The organizations are:

Oregon Mathematics Education Council
Oregon Council of Teachers of Mathematics
Oregon Math Leaders
Teachers of Teachers of Mathematics
Community College Math Chairers
Honors Institutes

For additional copies of The Oregon Mathematics Concept Papers, or information regarding speakers and/or discussion leaders, contact:

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The Oregon Department of Education supports, facilitates and coordinates ongoing efforts among these organizations, including the OMEC Curriculum Project. Don Fineran serves as Department liaison to all of the above organizations.

The OMEC would like to thank Al Elkins of Porter Public Relations, Portland, for his help in editing these papers.

OMEC TASK FORCE
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GEOMETRY, K-12

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INTRODUCTION

As a body of subject matter, geometry contains some of the most elegant, esthetically pleasing, and practical ideas in the mathematics curriculum. The study of geometry is visual as well as analytic, integrating spatial relations and number concepts. As the first branch of mathematics to be formalized, thanks to Euclid, geometry has occupied a revered place in the mathematics curriculum, even in Western culture. Yet, the study of geometry is a source of frustration and bewilderment for many students.

Several recent research studies at The University of Chicago, Brooklyn College, and Oregon State University on teaching and learning geometry have helped explain the difficulty that students have with geometry. First, geometry concepts seem to be learned, like number concepts, in stages, with development progressing from concrete to abstract levels. Second, considerable time must be spent in developing students' understanding of the ideas in geometry, beginning in the early grades. For many students, this development may require years, rather than months of study. Third, the emphasis on formal mathematical proof is inappropriate for all but a relatively small minority of secondary school students. The study of the concepts of geometry, problem solving in geometry, and applications of geometric ideas and principles, however, seems appropriate for nearly all students.

The interrelationships between algebra and geometry must never be neglected in any treatment of secondary school geometry. Geometry provides an environment for picturing many algebraic results in one, two, and three dimensions. In particular, coordinate geometry, geometrical interpretations of formulas, pictures of figurate numbers, and graphs of functions, equations, and inequalities are all geometrical representations of algebraic concepts. A sequence of courses that interposes the study of geometry between algebra courses fosters interaction of algebraic and geometric concepts, problem solving experiences, and applications.

With these thoughts in mind, the OMEC Geometry Task Force has prepared recommended curriculum topics and materials for the study of geometry at the following levels:

Grades K-3
Grades 4-6
Middle School
High School

These recommendations are based on successful practices of successful teachers, both members of the task force and their colleagues around the state. In addition, several recommendations for teacher education are included.

Recommendations at the high school level are split into two suggested areas: Integrating Informal Geometry and Proof in High School and Applied Geometry in High School (excluding formal proof). The Integrated Geometry course outline is based on a course at Catlin Gabel School in Portland, while the Applied Geometry course outline is based on a course at Medford High School in Medford.

References

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Additional references appear in the course outlines and at the very end of this report.

TOPICS AND ACTIVITIES FOR GEOMETRY, K-3

Topics

1. Identify basic spatial relationships: over/under/on, above/below, near/far, high/low, inside/outside, right/left, up/down, first/last, open/closed, etc.

2. Distinguish between straight and curved lines, flat and curved surfaces, open and closed objects.

3. Understand simple types of symmetry in shapes and in nature.

4. Recognize and represent various types of simple planar shapes: e.g., triangles, squares, rectangles, circles.

5. Recognize and name various types of simple solid shapes: e.g., cubes, cylinders, cones, spheres, pyramids.

6. Identify simple properties of the shapes in 4 and 5 above, using relationships in 1, 2 and 3.

Activities

(Refer to paragraphs 1 through 6 in the Topics.)

For Topic:

1. a. Manipulate blocks, boxes and other shapes.
   b. Make drawings.
   c. Sort attribute blocks and other blocks.
   d. Trace and color shapes.
   e. Act out relationships.

2. a. Make straight/curved lines and formations of objects (blocks, people.)
   b. Draw shapes and lines.
   c. Build towers and stacks of blocks.
   d. Roll shapes.

3. a. Fold and cut paper patterns.
   b. Make pattern block and mirror patterns.
   c. Look for symmetry in nature: leaves, flowers, animals, human body.
   d. Finish partial drawings: e.g., given the right half, draw the left half.
   e. Fold paintings to make symmetrical paint blots.
   f. Identify symmetry in clothing, carpeting, linoleum, quilts.

4. a. Copy shapes on geoboards and dot paper, with templates and cutouts.
   b. Draw and cut out varieties of shapes.

c. Produce a variety of single types of shapes, using geoboards, dot paper, drawings, and cutouts. (For example, large triangles, small triangles, "fat," "skinny," turned, etc. triangles.)
d. Solve puzzle activities: tangrams, pentominoes, etc.
e. Find hidden figures in drawings.
f. Dissect shapes into other shapes: e.g., squares into four triangles.

5. a. Sort geoblocks and other solid shapes.
b. Build shapes from Tinkertoys, straws, clay, paper, blocks, etc.
c. Draw faces of simple shapes.
d. Count faces, edges, and vertices of solid shapes.
e. Determine which shapes would hold liquid.

6. a. Make sortings of shapes according to properties.
b. Count vertices, diagonals, angles, faces and edges.
c. Compare and contrast according to their properties. (How are they alike? How are they different?)
d. Make "clubs" of shapes according to their attributes. Which shapes belong to several clubs?
e. Make "turtle" drawings of shapes in LOGO.

Materials

Pattern blocks
Shape cutouts
Tangrams
Pentominoes
Geoblocks and other solids
Polyhedra models
Sticks, straws
Tinkertoys
D-Stix
Paper for folding
Mirrors
Photographs and professional drawings
Dot Paper
Geoboards
Templates
Flowers, leaves
Computers, LOGO

TOPICS AND ACTIVITIES FOR GEOMETRY, 4-6

Topics

1. Identify varieties of shapes and solids according to relevant properties.

2. Analyze shapes according to properties of sides, angles, diagonals, faces, edges, etc.

3. Use basic drawing and measurement instruments (e.g., ruler and protractor) and templates to draw shapes and patterns of shapes.

4. Analyze shapes and solids according to symmetry properties (reflection, rotation).

5. Perform basic constructions with compass, Mira and ruler.

6. Draw views of solid shapes (top, front, side).

7. Visualize and identify shapes given three views.

8. Use simple coordinate geometry (graph paper geometry) to draw two- and three-dimensional shapes.

9. Draw simple shapes and patterns using "turtle geometry" (LOGO).

Activities

(Refer to paragraphs 1 through 9 in the Topics.)

For Topic:

1. a. Sort shapes in as many ways as possible according to properties of sides, angles, diagonals, faces, edges, etc.
   b. Represent shapes on geoboards and dot paper.
   c. Make varieties of shapes using pattern blocks and tangrams.
   d. Make patterns to fold up into solid shapes - cubes, prisms, pyramids, cylinders, etc.
   e. Identify hidden shapes in picture puzzles.
   f. Identify mystery shapes according to clues (properties) revealed in sequence. Justify the identifications using the properties.
   g. Identify two- and three-dimensional shapes in nature, in architecture, in cloth and floor covering patterns, etc.

2. a. Analyze shapes on dot paper and geoboards using lengths of sides, angle measure, lengths of diagonals, angles between diagonals, etc. Use measuring devices if necessary (ruler and protractor). The dot paper structure reveals many properties and minimizes the need for measuring instruments.

b. Make "clubs" of two- and three-dimensional shapes according to necessary properties. Determine which shapes belong to several clubs. For example, squares belong to the rectangles club, the parallelograms club, and the rhombi club (maybe to the trapezoid club as well, depending on membership criteria).

c. Make bulletin board displays of shapes and their properties using "sometimes/always/never" qualifications. (For example, a parallelogram sometimes has all sides congruent, always has opposite sides which are parallel, and never has more than four sides.) Display two- and three-dimensional shapes.

d. Use paper-folding activities to discover various properties of shapes.

3. a. Draw varieties of shapes ("large," "small," "fat," "thin," turned, etc.) Write a story about your shapes.

b. Use pattern blocks to make interesting patterns. Draw your pattern and decorate it.

c. Draw enlargements and reductions of shapes, keeping the same proportions (i.e., simple shapes).

d. Draw faces of three dimensional shapes.

4. a. Use mirrors (or Miras) to find lines of symmetry in two dimensional shapes. Use tracings to find centers of rotation symmetry.

b. Find symmetry properties of flowers, leaves, animals.

c. Identify symmetry properties in patterns of shapes, as in floor coverings, wallpaper, cloth, brick patterns, etc.

d. Complete an arrangement of shapes to give it symmetry properties.

e. Use pattern blocks and mirrors to generate patterns with rotation symmetry (e.g., church window).

f. Find planes of symmetry in prisms, cylinders, cones, spheres.

5. a. Perform various basic constructions using ruler and compass or Mira and compass. Copy line segments, angles, circles, triangles. Bisect line segments and angles. Construct perpendiculars and parallels, etc.

b. Construct shapes inscribed in circles and vice versa.

c. Construct designs using basic constructions. Make bulletin board displays of decorated designs.

d. Make simple house plans using construction instruments.

6. a. Draw views of three dimensional shapes including stacks of cubes.

b. Draw views of shapes and stacks of shapes from memory.

c. Make oblique drawings of prisms, pyramids, cones. Use dot or graph paper if necessary.

d. Make perspective drawings of simple shapes: e.g., cubes.

7. a. Select correct views of shapes from sets of possibilities.

b. Select correct shape from sets of views.

c. Identify simple cross sections of solid shapes.

d. Describe solid shapes made by spinning simple two-dimensional shapes.

8. a. Use positive and negative coordinates to graph shapes.

b. Make up patterns of shapes and read coordinates of vertices.

c. Investigate effects of drawings by varying coordinates: e.g., negating one or both coordinates, multiplying one or both coordinates by a constant, adding constants to one or both coordinates.
d. Use coordinates to make symmetry patterns, enlargements and reductions.

9. a. Draw simple two-dimensional shapes using turtle graphics.
b. Combine shapes to make pictures.
c. Use 'repeat' commands to make patterns of shapes.

Inservice Opportunities

Math for the Curious Workshop
OCTM inservice workshop and conference
THE OREGON MATHEMATICS TEACHER articles
Math Their Way conferences and workshops
Northwest Mathematics Conference
NCTM conferences and publications: e.g., THE ARITHMETIC TEACHER

MIDDLE SCHOOL GEOMETRY

1. Goals and Objectives

    a. Abstraction of concepts
    b. Establish relationships among geometric properties informally
    c. Infer properties from structures (dot paper, graph paper, tessellations, etc.)
    d. Measure abstractly

2. Prerequisite Skills

    a. Recognition and identification of two- and three-dimensional shapes
    b. Description and differentiation of shapes by attributes
    c. Measurement in concrete terms (using measuring devices and standard systems of units)

3. Rationale

    a. Content

    - Students have a natural curiosity about and interest in geometrical ideas.
    - Geometry is closely related to the students' world, since it is the study of space and shapes around them.
    - Geometrical ideas are applied in many areas of science and technology. Many of these ideas can be applied in industrial arts, science and mechanical drawing classes, but should be introduced in mathematics class.
    - Geometry is a unifying theme of mathematics. With a background in geometry, students have an alternate, visual way of viewing topics in mathematics.
    - Geometry can involve students in active inquiry, imaginative thought, discovery of relationships, the testing of conjectures, and critical, analytical reasoning.

    b. Manipulative Materials

    "Mathematics textbooks usually provide for sequential development of mathematical principles, concepts and skills, but often are weak in relating to concrete illustrations from the students' world of experience." (From Math Lab Junior High by McFadden, Anderson and Schaaf - Action Math Associates, Inc., 1975, Eugene, Oregon.) The use of manipulatives at all levels and the discovery approach adds to students' understanding and builds a basis for more abstract concepts.

    c. Approach

    - The study of geometry should be related to the students' world.
    - The geometry taught should fit the thinking and learning patterns of the students.

- The geometry should be informal, stressing concepts, applications, and problem solving, not a formal study of postulates, theorems and proof.
- The geometry should be mathematically cohesive, illustrating the interrelationships of properties and results. It should not be presented in disjointed pieces and should be far more than fun and games.

4. Topics and Activities

a. Lines

- Draw open and closed figures using line segments and curved lines.
- Identify parallel, skew, and intersecting lines in the environment.
- Identify perpendicular lines and right angles in the environment.
- Identify angles and rays in the environment.
- Illustrate on a geoboard or dot paper:
    - i. Intersecting, parallel, and perpendicular line segments.
    - ii. Right, acute, and obtuse angles.
- Identify complementary and supplementary angles.

b. Symmetry

- Identify reflectional, rotational and translational symmetry.
- Make symmetrical designs on dot paper or a geoboard.
- Explore motion geometry using manipulatives and LOGO.

c. Similarity and congruence

- Copy a drawing on squared paper onto a sheet with smaller or larger squares.
- Sort a set of figures into subsets of similar figures.
- Use a geoboard or dot paper to construct polygons with the same shape or the same size and shape.
- Recognize and use the terms "congruent" and "similar."
- Make enlargements and reductions using rectangular coordinates.
- Sketch and construct drawings congruent or similar to certain plane figures.
- Explore similarity and magnification using LOGO.

d. Tessellations

- Cover a surface without leaving gaps or overlapping using congruent polygons.
- Tessellate space regions using cubes, rectangular prisms, triangular prisms, etc.
- Select shapes that will or will not tessellate the plane.
- Use squared and isometric paper to create simple tessellating patterns and designs.
- Copy tessellations and designs using ruler and compass.

e. Plane Figures

- Sort geometric figures into related groups and name the groups.
- Sort geometric figures into specific groups according to specific properties.
- Identify concave and convex polygons.
- Identify and name circles, triangles, rectangles, rhombi, parallelograms, trapezoids, pentagons, hexagons and octagons.
- Give examples and state properties of regular polygons.
- Classify and name triangles according to properties of sides (scalene, isosceles, equilateral), angles (right, obtuse, acute), and combinations (right scalene, obtuse isosceles, etc.)
- Identify types of quadrilaterals and name their distinguishing properties: square, rectangle, rhombus, parallelogram, kite, trapezoid.
- Form families of quadrilaterals according to shared properties.

f. Constructions

- Use a geoboard and dot paper to make triangles, quadrilaterals, pentagons, hexagons, octagons, etc.
- Use LOGO to draw polygons.
- Use paper folding to make perpendicular and parallel lines and various polygons.
- Use compass and straightedge to copy and bisect a line segment, copy and bisect an angle, construct perpendicular and parallel lines, construct regular polygons, and make geometric designs.

g. Three-Dimensional Geometry

- Sort and classify three dimensional solids according to properties.
- Identify cubes, cones, cylinders, pyramids, and prisms in the environment.
- Select nets (patterns) that will or will not fold into cubes, prisms, and pyramids.
- Construct polyhedra by using stacks of cubes or blocks, ready-made patterns, or nets.
- Make a stack of cubes, given top, front, and side views.
- Make a drawing of a stack of cubes and simple polyhedra given top, front, and side views.

h. Linear and Angle Measurement

- Produce line segments of given length.
- Select the most appropriate unit of linear measure.
- Select a unit to obtain a reasonable degree of precision.
- Convert among customary (English system) units.
- Convert among metric units.
- Give approximate conversions between customary and metric units.
- Determine the perimeter of a polygon and circumference of a circle.
- Find distances indirectly by using scale drawings and maps.

- Use a protractor to measure angles in various polygons.
- Determine the sum of the angles in triangles, quadrilaterals, pentagons, hexagons, and octagons.

i. Area Measurement

- Determine the area of a rectangle by covering with squares.
- Use acetate grid paper to estimate and measure the area of polygons and irregular figures.
- Make polygons of a given area on a geoboard.
- Discover formulas for finding the areas of triangles, rectangles, and parallelograms.
- Find the surface areas of prisms and pyramids with given dimensions.

j. Volume and Capacity

- Find the volume of prisms using cubes.
- Find capacity using liquid measurement or dry material (rice, popcorn, styrofoam "peanuts," etc.)
- Convert among volume units within the English and metric systems.
- Find the volume experimentally of prisms, pyramids, cylinders, and cones.
- Discover a relationship between the volume of a pyramid (cone) and the volume of the smallest prism (cylinder) containing it.

5. Resources

In addition to the references at the end of this report, the following resources and materials are recommended:

Visual Thinking Cards. Dale Seymour Publications
Wooden Geometric Solids - Elementary Set (Grades 4-8) and Secondary Set (Grades 8-12), Creative Publications
Thinking Visually. Robert McKim, Creative Publications
Aftermath Series. Dale Seymour, Mary Laycock, Verda Holmberg, Ruth Heller, and Bob Larson, Creative Publications

INTEGRATING INFORMAL GEOMETRY AND PROOF IN HIGH SCHOOL

RATIONALE: The emphasis of this course is in a visual approach to problem solving in geometry. Two-thirds of the year is devoted to the development of informal geometric principles, and the last third is devoted to formal deductive reasoning and geometric proof.

Much evidence has appeared in recent years for the need to relate the study of geometry with other disciplines and to the physical world. The increasing necessity for students to acquire stronger visualization and conceptual skills is apparent in many ways. In geology and chemistry, for example, students need the ability to perceive and analyze symmetry. Arguments such as these have been influential in the development of the course described below.

1. Goals for the Course

    a. To increase students' skills in conceptualizing two- and three-dimensional shapes and in visual thinking.
    b. To improve students' understanding of spatial relations.
    c. To instill a positive attitude for and appreciation of geometry.
    d. To recognize and understand the role of geometry in describing our environment.
    e. To develop students' understanding of the historical and cultural aspects of geometry.
    f. To expose students to non-Euclidean geometries.
    g. To develop students' understanding of the relations between one, two, and three dimensions, and to explore higher dimensions.
    h. To enable students to draw simple two- and three-dimensional diagrams.
    i. To enable students to work comfortably with geometric models.

2. Topics

    a. Visual experiences and activities with two- and three-dimensional shapes; classifying shapes.
    b. Two-dimensional and three-dimensional drawing.
    c. Perspective drawing (one point, two point, more).
    d. Transformations in the plane.
    e. Constructions and their historical development.
    f. Lines, parallel and perpendicular lines, angles.
    g. Measuring length, angles, areas, volume.
    h. Triangles
        - Congruence
        - Similarity
        - Trigonometry of right angles
        - Inequalities

3. Use of Manipulatives/Environments

a. Activities and materials that enhance visual thinking
   - For transformations in the plane: construction instruments, including MIRAs; tracing paper
   - Geometric solids and models
   - Twenty questions game
   - Designs, trademarks, etc. in the real world that contain symmetry
b. MATH-AID templates from Australia. These templates are highly versatile in drawing various geometric figures.
c. Polyhedra modes from soda straws, paper, glass, D-Stix, or other kits.
d. Geoboards, dot paper (square and triangular), graph paper
e. Tangrams, Soma cubes
f. Optical illusions, Escher-type drawings
g. Construction and drawing instruments
h. Clinometer for trigonometry investigations
i. Computers
   - LOGO for turtle graphics, tessellations, drawings.
   - Geometric Supposer (Sunburst Communications) for formulating and testing conjectures about two dimensional shapes.
j. Films, videos, and pictures
   - MATHEMATICS OF THE HONEYCOMB
   - DONALD DUCK IN MATHEMAGIC LAND
   - THE CUBE
   - Nova series videos, FORM IN NATURE and A MATHEMATICAL MYSTERY TOUR
   - Slides and photographs of real world geometric shapes and objects: buildings, flowers, bridges, tilings, patterns (Alhambra drawings), logos, art, architecture, etc.

4. Project Ideas

Research projects can be an important part of the course. Individual students can investigate a topic of interest to them and present it to the class. Projects can include geometrical content; development of ideas, including building a model or making a poster and a paper describing the mathematics of the project; and a presentation to the class or teaching a short unit on the topic.

Suggested student project topics:

Curve stitching
Measurement and approximation
Map projections
Paper folding
Paradoxes and fallacies
Optical illusions
Approximation of pi
Pythagorean theorem proofs
Pythagorean triples
Finite geometries
Non-Euclidean geometries
Topology
Mathematics and art
Inversive geometry

Space filling curves
Figurate numbers
Golden section
Trisection of an angle
Nine point circle
Geometry of complex numbers and operations
Curves of constant diameter
Four-color problem
Fourth dimension
Koenigsberg bridge problem
Geometric constructions: possible and impossible
Polyhedral models

Conic sections
How Erastosthenes measured
the earth
Measuring stellar distances
Mathematics of a sundial
Mathematics of photography
Geodesic domes
History of measurement

Tessellations of the plane
Optics and the mathematics
of light rays
Acoustics and the mathematics
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Stonehenge
Soap films
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In addition to the references at the end of this report, the following are recommended:

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APPLIED GEOMETRY IN HIGH SCHOOL

1. Goals for the Course

   a. To develop appreciation of the relationship between the physical world and the abstract world of geometry
   b. To understand geometrical vocabulary and symbolism
   c. To develop awareness of vocational applications of geometry
   d. To develop and reinforce specific skills in:
      - Using special equipment associated with geometry
      - Reading and writing geometric results
      - Using coordinate geometry to solve problems
      - Applying geometric concepts and techniques to solve "real" problems
      - Manipulating algebraic expressions
      - Visualizing two-dimensional and three-dimensional relationships

2. Content

   a. Introduction to geometric ideas
      - Undefined terms and basic definitions
      - Postulates and theorems
      - Subsets of a line (segments and rays) and distance
      - Angles: adjacent, vertical, measuring angles, adding and subtracting angles, classification of angles, complementary and supplementary angles
   b. Perpendicular and parallel lines and planes, angle relationships, and other conclusions. Uniqueness of certain perpendiculars and parallels.
   c. Triangles
      - Definitions and classifications
      - Angle sum and conclusions for triangles and other polygons
      - Congruence: definition and minimum requirements for congruent triangles
      - Isosceles and equilateral triangles
      - Inequalities in triangles
      - Applications
   d. Other polygons
      - Parallelograms: definitions and properties
      - Rectangles, rhombuses, squares: definitions and properties
      - Trapezoids: definitions and properties
      - Applications
   e. Similarity
      - Ratio and proportion: definitions, solving, properties of proportions
      - Similar polygons: definitions and applications
      - Similar triangles: necessary conditions, segments divided proportionally, applications
   f. Right triangles
      - Radicals
      - Geometric mean
      - Pythagorean theorem: 30-60-90 triangles, 45-45-90 triangles

  - Diagonals of a prism
  - Applications
- g. Right triangle trigonometry
  - Tangent ratio
  - Sine and cosine
  - Applications
- h. Circles
  - Basic definitions, properties of circles and lines
  - Tangents
  - Arcs and central angles
  - Arcs and chords
  - Inscribed angles, other angles
  - Applications
- i. Area and volume
  - Areas of polygons: rectangles, parallelograms and triangles trapezoids, regular polygons
  - Circles: circumference and area
  - Solids: surface area and volume of right prisms, pyramids, circular cylinders and cones
  - Applications
- j. Coordinate geometry
  - Points in the plane
  - Distance, midpoint, slope, parallel and perpendicular lines
  - Equations of a line
  - Applications
- k. Constructions and loci
  - Segments, angles, and their bisectors
  - Perpendiculars and parallels
  - Circles
  - Special segments
  - Describing a locus
  - Intersections of loci
- l. Deductive reasoning
  - History of Euclidean geometry
  - If-then conditionals
  - Two column proof

3. Application Materials

Special packets of application materials have been developed for each content unit. These are designed to familiarize students with applications of geometric concepts in industrial, technological, and vocational settings, as well as to develop geometric problem solving skills. (Sample packets are available from Gary Morrison at South Medford High School.)

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TEACHER EDUCATION RECOMMENDATIONS

In order to appreciate the importance of geometry in the curriculum and to teach geometry effectively, it is imperative that new teachers in Oregon study a substantial amount of geometry. For prospective elementary teachers, at least one full quarter of informal/investigative geometry is recommended, including the study of two- and three-dimensional shapes, patterns of shapes and symmetry, measurement, constructions, motion geometry, congruence and similarity via transformations, coordinate geometry, and problem solving in geometry. An accompanying course in LOGO is recommended. Ideas and materials for teaching elementary geometry concepts, such as those in the ***Mathematics Resource Project*** (Creative Publications) and ***Problem Solving in Mathematics*** (Dale Seymour Publications), should be emphasized in methods courses.

For prospective middle school and high school teachers, the study of informal geometry concepts and problem solving in geometry is recommended. In addition, the study of Euclidean, non-Euclidean, and transformational geometry is urged. Courses in LOGO, emphasizing problem solving are encouraged. The uses of computers in geometry are appropriately studied in both methods and content courses. Software packages, such as The Geometry Supposer, should be demonstrated and analyzed for their usefulness in the geometry classroom.

GEOMETRY FOR OREGON SCHOOLS
APPENDICES

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REFERENCING INFORMAL GEOMETRIC TOPICS WITH TEACHING ENVIRONMENT

ENVIRONMENT TOPIC	GRAPH PAPER	GEOBOARDS/ DOT PAPER	BLOCKS CUTOUTS SHAPES	DRAWINGS/ CONSTRUCTIONS	PAPER FOLDING	LOGO*
VISUALIZATION	8 16	16	16 40 43 48	4 16 43	18 38	2
PROPERTIES OF 2D SHAPES	3 16 41	3 13 16 19 28 29	3 14 16 20 25 39 40 44 45 48 52 53 56	11 16 25 32 49 55	38	1 2 31 34 35 46 59
TESSELLATIONS	8 16	3 5 16	3 16 47	4 5 42		2 31 34 35 46 59
MOTIONS AND SYMMETRY	8 16	13 16	16 21	11 17 26 32 33	38	2 34 46
MAGNIFICATION AND SIMILARITY	8 16		16	16		2 31 34 35 46 59
3D SHAPES/ POLYHEDRA	16		16 21 27 40 47 51	4 15 16	16 18	2
GEOMETRICAL DESIGN	6 7 16 41	6 16 50	39 42 47 52	4 6 11 15 17 26 30 42 49 50 55	16 18 38	1 2 31 34 35 46 59
MEASUREMENT	16	13 16 19	16 40			34

*Ideas for LOGO projects can also be found in 5, 6, 14, 17, 20, 25, 39, 42, 44, 45, 48, 52, 56.

Oregon Mathematics Concept Paper No. 4

STATISTICS AND PROBABILITY, K-12

PREFACE

The Oregon Mathematics Education Council (OMEC) launched a mathematics curriculum project in May 1984. Nearly 100 enthusiastic, dedicated volunteers, a majority of whom are classroom teachers, worked on the task forces of the project. These groups worked to develop a set of concept papers relating to major issues of reform in mathematics education. Their ideas are shared now as a set of The Oregon Mathematics Concept Papers. They are available as a complete set or as separate papers. You are encouraged to read those papers of special interest and to discuss the ideas with colleagues. Duplicating the papers for such noncommercial purposes is encouraged.

OMEC is a widely representative advisory group, representing teachers at all school levels, counselors, administrators, school boards, business, labor, industry, and parents. The Oregon Mathematics Concept Papers are being disseminated by the Oregon Mathematics Project, working with a number of mathematics organizations, and many individuals. The organizations are:

Oregon Mathematics Education Council
Oregon Council of Teachers of Mathematics
Oregon Math Leaders
Teachers of Teachers of Mathematics
Community College Math Chairers
Honors Institutes

For additional copies of The Oregon Mathematics Concept Papers, or information regarding speakers and/or discussion leaders, contact:

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The Oregon Department of Education supports, facilitates and coordinates ongoing efforts among these organizations, including the OMEC Curriculum Project. Don Fineran serves as Department liaison to all of the above organizations.

The OMEC would like to thank Al Elkins of Porter Public Relations, Portland, for his help in editing these papers.

OMEC TASK FORCE
on
STATISTICS AND PROBABILITY, K-12

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INTRODUCTION

Statistics and probability are two subjects rapidly gaining importance and credibility as strands of study throughout the mathematics curriculum K-12. Many educators have had a minimal amount of formal exposure to these topics. Staff development will be necessary to incorporate more statistics and probability into the current teaching of mathematics. Educators (teachers and administrators) needing to become comfortable with teaching these subjects should take advantage of all available workshops and inservices. For further information, contact may be made to:

Oregon Department of Education (Don Fineran),
Oregon Council of Teachers of Mathematics (OCTM), or
Teachers of Teachers of Mathematics (TOTOM)

An annotated bibliography and a sample lesson have been provided in this report. Please take the time to do some reading and try a number of activities. Teaching statistics and probability can also be an excellent way to review basic skills in whole numbers, fractions, decimals and/or percents. Of utmost importance, do not be afraid to jump in and "get your feet wet." Both you and your students can have a great deal of fun with these subjects!

STATISTICS AND PROBABILITY

RATIONALE

"It is truth very certain that, when it is not in one's power to determine what is true, we ought to follow what is more probable." Rene Descartes

Statistics is the science or study of data. Statistical studies require collecting, sorting, representing, analyzing, and interpreting information. That information is used for predicting, drawing inferences, and making decisions. There is always some degree of uncertainty with data that is collected. This means that statistics is most often concerned with using information in the face of uncertainty. Probability gives a way to measure uncertainty and is therefore essential to understanding statistical methods.

WHY TEACH STATISTICS AND PROBABILITY?

Recent technological advances have made more statistical information available to us than ever before. In conversation, on television and in newspapers, we are flooded with data that demands an increasing understanding of statistics and related processes. What is the effectiveness of seat belts and should you wear them? What can you do to protect yourself against the rising cost of living? Is there a justifiable connection between smoking and disease? Which money market or retirement fund makes most sense? What does a particular advertising claim really mean? Is the death penalty a deterrent to crimes of murder? How do you weigh the benefits of birth control pills against their causing cancer in rats under some conditions? What effect will the rate of population growth have on our lives? How should the increased number of older people in our society affect social decisions? For all of these questions, and many, many others, statistics and probability concepts are fundamental to making decisions.[1]

Studying statistics and probability also helps develop students' critical thinking skills. In carrying out experiments in statistics and probability, students develop ways to cope with uncertainty as they are searching for the truth in a situation and learning to report it faithfully. Approaching situations statistically can make a student face up to prejudices, think more consistently about arguments, and justify their thinking with numerical information. This has applications in all areas of life--social, political, and scientific.[1]

In the past, the study of probability in secondary school has almost of necessity concentrated on formulas and theoretical principles. Situations were simulated by means of spinners, tossing dice and coins, and drawing colored balls out of an urn. While simulations of this sort can be very useful, making a large number of replications is time consuming. Introducing a micro-computer into a classroom allows probability to be studied in a new and exciting way. Simulation can become an important mathematical technique for the students. An experiment can be replicated a large number of times to determine an empirical probability. This empirical probability can provide a check on the theoretical analysis of the problem. In fact, the analysis involved in writing a simulation may form a basis for a later proof.[2]

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[1] Marilyn Burns, PROBLEM SOLVING: The Math Challenge of the '80s, Learning Institute

Since most schools have access to calculators and computers, teachers should know the capabilities of these devices and how to take advantage of them as powerful teaching tools in the study of statistics and probability. As with micro-computers, calculators can facilitate instruction in statistics and probability because numbers need not come out nicely in order to perform the calculations required. Experimental data can be used more effectively and data can be more realistic when calculators are available to aid calculation.

INSTRUCTIONAL PROCESS

In the development of mathematical concepts, learning is an active process in which two characteristics are common:

1. Development begins at the concrete level, using manipulatives and building on remembered experiences, and moves gradually through a representative level to the abstract level.

2. The problem-solving approach, sometimes utilizing guided discovery, and sometimes including independent investigations, is used extensively in the development of concepts and skills.

INSTRUCTIONAL GOALS: STATISTICS AND PROBABILITY

We have organized the study of statistics and probability into five major goals.

The learner will demonstrate the ability:

1. To collect and record appropriate data in a given situation.

2. To organize data and make accurate representations of the data in tables, charts or graphs.

3. To develop and use sampling techniques.

4. To use various techniques to represent possible outcomes of an experiment and assign reasonable probabilities to these outcomes.

5. To interpret data, to make comparisons, to test predictions, and to draw inferences.

NOTE: These goals are meant to provide minimal core instruction in statistics and probability. Goals are listed at the mastery level, but individual districts may choose to introduce them at an earlier level.

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[2] Glenda Lappan & M.J. Winter, PROBABILITY AND COMPUTERS: Simulate, then Analyze, Michigan State University.

GOAL 1:

The learner will demonstrate ability to collect and record appropriate data in a given situation.

Students will:

K - 1	2 - 3	4 - 6	7 - 9	10 - 12
• Gather factual data from personal experiences and environment.	—	—	—	—|
• Use physical objects to record data.	—|	• Gather and record factual data from variety of resources. - Integrate with other subject areas.	—	—|
• Tally by 1's.	• Tally by 5's (~~1111~~ 1)			
		• Collect data from probability experiments.	—	—|
			• Use the computer to generate data.	—|

GOAL 2:

The learner will demonstrate ability to organize data and make accurate representations of the data in tables, charts, graphs, or diagrams.

Students will:

K - 1	2 - 3	4 - 6	7 - 9	10 - 12
• Sort and classify people and objects.				
• Make real graphs which compare groups of objects.			• Select and construct appropriate display of data, e.g., - Line graph - Bar graph - Box plots - Circle graph - Pictograph - Venn diagrams	
• Make picture graphs which use models to stand for real things.				
	• Construct bar graphs.			
		• Organize information into tables/charts and diagrams given appropriate scale: - Box Plots - Line plots - Venn diagrams	• Students determine scale.	
		• Construct line graphs.		
		• Use computer generated graphs and tables.		
			• Use the computer to generate graphs and tables.	

GOAL 3:

The learner will demonstrate ability to develop and use sampling techniques.

Students will:

K - 1	2 - 3	4 - 6	7 - 9	10 - 12
• Take samples from classroom members and family.				
	• Take non-random samples from larger groups such as school population.			
			• Design surveys.	
		• Take random samples from the community.		
		• Understand the relationship between size of sample and degree of certainity.	• Analyze factors which may cause data to be a poor predictor.	
		• Understand biases involved in sampling techniques.		
			• Realize that surveys show tendencies - not exact results.	
				• Design and conduct simulations.

GOAL 4:

The learner will demonstrate ability to use various techniques to represent possible outcomes of an experiment and assign probabilities to these outcomes.

Students will:

K - 1	2 - 3	4 - 6	7 - 9	10 - 12
• Experience activities which have outcomes that depend on chance. →	→	• Understand fair and unfair games for experimentation.	• Design or revise a game to make it fair. →	—|
	• Experience activities with non-chance outcomes.		• Use intuitive counting techniques. →	• Use combinations and permutations when appropriate.
	• Make informal predictions - - possible/impossible - likely/unlikely - certain/uncertain.		• Understand the difference between situations where you can/cannot dedetermine theoretical probabilities, e.g., - Roulette wheels vs. life insurance tables. →	• Define probabilities based on relative frequencies for which no theoretical model exists.
			• When appropriate, determine experimental probabilities where a theoretical model does exist. →	—|

GOAL 4: (Continued)

The learner will demonstrate ability to use various techniques to represent possible outcomes of an experiment and assign probabilities to these outcomes.

Students will:

K - 1	2 - 3	4 - 6	7 - 9	10 - 12
		• Understand meaning of probabilities of 0 (impossible) and 1 (certain).	• Describe the range of probability from 0 to 1.	
			• Be able to represent probabilities in fraction, decimal and percent form.	⟶|
		• Use charts, tables, and lists to organize all possible outcomes of an experiment.	• Know various ways to organize information to analyze all possible outcomes of an experiment; e.g., - Tree diagrams - Area models - Pascal's triangle.	⟶|
			• Use a theoretical model to assign probabilities based on analyzing all possible outcomes of an experiment.	→ • Use the computer to check the theoretical analysis of a problem.
			• Compare experimental probabilities with their theoretical model.	⟶|

GOAL 4: (Continued)

The learner will demonstrate ability to use various techniques to represent possible outcomes of an experiment and assign probabilities to these outcomes.

Students will:

K - 1	2 - 3	4 - 6	7 - 9	10 - 12
			• Understand the differences in events which are: - Equally likely/ not equally likely. - Dependent/ independent.	
				• Use the (micro) computer to simulate experiments and assign reasonable probabilities to the outcomes, when no theoretical probability model exists.
			• Determine the ratio between the probability for and the probability against an event occurring (odds).	

GOAL 5:

The learner will demonstrate ability to interpret data to make comparisons, predictions, and inferences.

Students will:

K - 1	2 - 3	4 - 6	7 - 9	10 - 12
• Read and analyze information on graphs to answer comparison questions.				
	• Pose questions which can be answered using data.		• Identify unresolved questions.	
	• Draw conclusions based on organized data.			
		• Understand the concept of and determine the arithmetic mean. →	• Understand the concepts of and determine median, mode and range. →	• Determine quartiles and percentiles.
			• Know how measures of central tendency such as mean, median and mode, i.e., "average", are used to facilitate a variety of interpretations.	
			• Understand what is meant by a "normal distribution."	
				• Understand the concept of a random variable.
				• Use random variable to generate a probability distribution.

GOAL 5: (Continued)

The learner will demonstrate ability to interpret data to make comparisons, predictions, and inferences.

Students will:

K - 1	2 - 3	4 - 6	7 - 9	10 - 12
				• Define mean and variance (standard deviation) of a random variable.
				• Use sampling techniques to approximate the mean and standard deviation of a random variable.
				• Use sampling techniques to test hypotheses about population statistics (mean, standard deviation).
			• Relate probability experiments to applications.	
	• Use intuition to make choices.	• Justify conclusions with intuition and reason.		
			• Use the results of empirical data to analyze reliability and make inferences.	

GOAL 5: (Continued)

The learner will demonstrate ability to interpret data to make comparisons, predictions, and inferences.

Students will:

K - 1	2 - 3	4 - 6	7 - 9	10 - 12
			• Solve problems and make decisions based on data.	
			• Understand ways statistics are used and misused to represent data in real-world situations.	

SAMPLE ACTIVITIES

"The Dice Experiment"

This activity may help students, grades K-12, in meeting five of the goals suggested in this paper, as follows:

Goal of Instruction	Abbreviation
G 1	"Collect"
G 2	"Organize"
G 3	"Sample"
G 4	"Assign Probabilities
G 5	"Interpret"

Level: K-1

Objective: Students will be able to represent on a graph the results of a die toss, and answer comparison questions.

Materials: Butcher paper graphs (see Figure 1), crayons, dice.

Procedure:

Goal 1, 2, 3

1. Each student rolls the die and records the roll on the butcher paper graph. This can be done with an "X" or by shading in the appropriate square (see Figure 1).

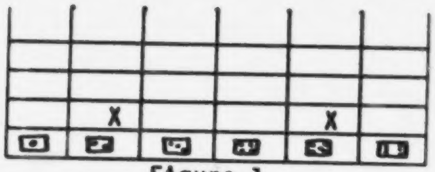

Figure 1

Goal 5

2. Comparison questions should be asked by the teacher.

   "Which one occurred the most? "Least?"
   "Are any the same?"
   "How many of each?" Etc.

3. Teacher asks class if they think they will get the same results if they repeat the experiment. Discussion.

Goal 1, 2, 3

4. Repeat the experiment in groups of 2-3. Roll 20 times and record on individual charts as in Figure 1.

Goal 5

5. Compare individual results with class graph.

6. Extend the activity. Repeat the experiment recording only every other toss. How do the graphs compare? Differ?

Level: 2-3

Objective: To present a probability experiment which compares theoretical and experimental probability.

Materials: Dice (1-6), copies of graph (see Figure 2).

Procedure:

Goal 1, 2

1. Predict which number will occur most often on a roll of one die. Construct a graph of the predictions (see Figure 2).

Goal 3

2. Partners roll a die 20 times and record their results.

3. Partners compare their predictions and results.

Goal 2

4. Class results are gathered and compared to the predictions.

Goal 5

5. Discuss outcomes.

   Some questions might include:

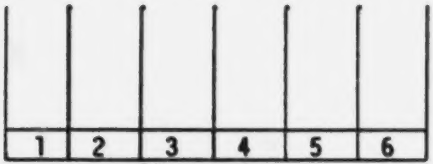

Figure 2

"Can you tell why you made your prediction?"
"Which occurred most often? Least often? Same?"
"Were there more 1's or 6's? 2's or 5's? 3's or 4's? . . ."
"How many more 6's (for example) than 2's (for example)?"
"How many altogether?"
"How would the results change if we rolled more times?"
"Do you think we would get the same results if we did it again?"

6. Extend the activity.

   Set up a station where partners can repeat the experiment.
   Try the same procedure with different numbered dice (e.g., 4-9).
   Try the same procedure with different shaped dice (e.g., octahedral, pyramidal).
   Try the same procedure with more rolls or more dice.
   Try the same procedure by subtracting the numbers rather than adding.

Level: 4-6

Objective: To present a probability experiment which compares theoretical and experimental probability.

Materials: 2 dice, recording paper.

Procedure:

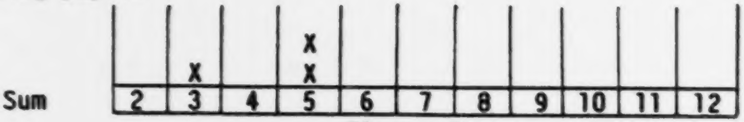

Figure 3

Goal 1, 2

1. Predict which sum(s) will occur most often when two dice are rolled and the numbers on top are added. Construct a graph of predictions (Figure 3).

Goal 3

2. Partners roll two dice 36 times and record the sums with an "X" on the graph (see Figure 3).

3. Partners compare predictions and results.

Goal 2

4. Class results are gathered and compared to predictions.

5. All possible outcomes are described (Figure 4) and organized (Figure 5).

Die 1

Die 2

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Figure 4

Sum	2	3	4	5	6	7	8	9	10	11	12
Occur	1	2	3	4	5	6	5	4	3	2	1

Figure 5

Goal 5

6. "What patterns do you see?"
"What sum could we expect to roll most often if we rolled many times?"
"How can we interpret our results?"

Goal 4

7. Probabilities are assigned to all outcomes.

P(2) = 1/36
P(3) = 2/36=1/18
P(4) = 3/36=1/12
P(5) = 4/36=1/9
P(6) = 5/36
P(7) = 6/36=1/6
P(8) = 5/36
P(9) = 4/36=1/9
P(10) = 3/36=1/12
P(11) = 2/36=1/18
P(12) = 1/36
P(13) = 0/36=0

8. Extend the activity.

Use a calculator to express the probability as a decimal or percent. Repeat the experiment using the product rather than the sum. (See level 2-3, procedure step 6 for other extensions.)

Level: 7-9

Objective: To present a probability experiment which compares theoretical and experimental probability.

Materials: Dice (4-9).

Procedure: Same procedure as level 4-6 except use 4-9 (or other) dice.

Goal 1, 2

1. Predict which outcomes will occur for a given operation for a particularly marked set (one, two or more) dice.

Goal 1, 3

2. Roll dice an agreed number of times and record the results.

Goal 5

3. Compare individual and class results to predictions.

Goal 2

4. Describe, organize and represent all possible outcomes (see Figure 3 and Figure 4 for grades 4-6).

Goal 4

5. Assign probabilities to all outcomes in fraction, decimal and percent forms.

Goal 3

6. Partners design and conduct a dice experiment following the same procedure.

Goal 5

7. Extend the activity.

   Analyze a state lottery for the chances of winning/losing.
   Examine applications of probability (e.g., games of chance, commercial applications, scientific or business applications).
   Discuss and write odds.
   Find various ways to represent data (Step 5).

Level: 10-12

Objective: To present a probability experiment which compares theoretical and experimental probability.

Materials: Dice, computer.

Procedure:

Goal 2, 3

1-5. Same procedure as level 7-9.

Goal 4

6. Determine odds.

Goal 1-4

7. Use the computer to establish a class data base for the dice experiment.

Goal 5

8. Answer various questions using the computer data base. Use your understanding of central tendency and variability to interpret the data.

   "What was the mean number of rolls made by the class?"
   "What was the median number of rolls made by the class?"
   "Do the results follow a normal distribution?"
   "How reliable are your results?"
   "State a question which can be answered by your data."

9. Use appropriate vocabulary and notation for probability study.

10. Extend the activity.

Goal 5

Analyze a game of chance.

Goal 2, 3

Design, conduct and analyze an experiment.

Goal 5

Investigate probability concepts in applications or situational lessons.

Write a program to simulate the results of rolling dice.

STATISTICS AND PROBABILITY

BIBLIOGRAPHY

Huff, Darrell. How To Lie With Statistics. New York: W.W. Norton and Company, Inc., 1954.

> This 142-page book is written at an intermediate reading level. There are many humorous examples of the misuse of statistics, as well as examples that demonstrate how statistics can be used to support any point of view. The examples of distortion and suggestions for examination are excellent.

Jacobs, Harold. Mathematics: A Human Endeavor. San Francisco: W.H. Freeman and Company, 1970.

> This 529-page hardbound textbook is written with a high reading level. This book contains three relevant chapters on methods of counting, introduction to probability and introduction to statistics. It is meant to be used as a resource book rather than as a beginning textbook. The examples presented are of a high motivational level for the advanced student in middle school.

Mathematics Resource Project. Statistics and Information Organization. Palo Alto, California: Creative Publications, 1977.

> An 888-page resource manual contains not only teacher direction, but many student worksheets which are reproducible. The material dealing with gathering information, tables, graphs, mean, median, mode, sampling, experimental probability, counting techniques, and applications in business, environment, medicine, culture and recreation. Also includes an extensive bibliography and glossary.

Middle Grades Mathematics Project. Michigan State University, Probability: Addison-Wesley, Inc., 1985.

Mosteller, Frederick, William H. Kruskal, Richard F. Link, Richard S. Pieters, and Gerald R. Rising, eds. Statistics By Example. Menlo Park, California: Addison-Wesley, Inc., 1973.

> A series of four books, each 150 pages with a high reading level. The books are: Exploring Data, Weighing Chances, Detecting Patterns, and Finding Models. Many situations are illustrated that explain real-life problems in terms of statistics. A knowledge of intermediate algebra is necessary to understand the mathematics involved.

Murphy, Elaine C. Developing Skills with Tables and Graphs, Books A and B. Palo Alto, California: Dale Seymour Publications, 1981.

> Two books of 53 pages with an easy reading level. Book A is designed for grades 3-5, Book B for grades 6-8. Most of the book consists of student worksheets beginning with tallies, tables, graphs and interpreting tables and graphs. There is a minimum of basic arithmetic. The pages are uncluttered and easy to read.

National Council of Teachers of Mathematics. Organizing Data and Dealing with Uncertainty. Washington, DC: The Council, 1979.

National Council of Teachers of Mathematics. Teaching Statistics and Probability. Washington, DC: The Council, 1981 Yearbook.

Many suggestions for content, projects and applications of statistics and probability at the K-12 level. Extensive bibliography.

Nuffield Mathematics Project. Pictorial Representation. New York: John Wiley & Sons, 1967.

Schools Council Project. Teaching Statistics. Berks, England: W. Foulsham & Company Ltd., 1980.

Excellent collection of 27 booklets for students 11-16 years old with an easy reading level. The books introduce basic principles of statistics and then apply these principles to many subject areas (health, career, science). Emphasis is on aspects of inference. Simulations, games, experiments and case studies are used in teaching statistical concepts.

Shulte, Albert, and Stuart Choate. What Are My Chances? Books A and B. Palo Alto, California: Creative Publications, 1977.

These books are 97 pages of a medium reading level. There are 73 activities to develop intuitive feeling about probability. Extensive teacher notes are included.

The Statistics Teacher Network. Ann Watkins, Dept. of Mathematics, Los Angeles Pierce College, 6201 Winnetka Avenue, Woodland Hills, California, 91371.

A newsletter published three times a year by the American Statistical Association & National Council of Teachers of Mathematics Joint Committee on the Curriculum in Statistics and Probability. Subscription is free upon request.

American Statistical Association & National Council of Teachers of Mathematics Joint Committee on the Curriculum in Statistics and Probability. Exploring Data, The Art and Techniques of Simulation, Introduction to Probability, Information From Samples, 1985.

This set of four books is written by James Landwehr, Ann Watkins, Mrudulla Gnanadesikan, Richard Scheaffer, Clarie Newman, Thomas Obremski and the guiding light, Jim Swift. Exploring Data concerns statistical techniques and includes practice in division, percents, ratios, etc. Tables, mean, scatter plots, stem and leaf plots, smoothing and data analysis are included. The Art and Techniques of Simulation shows how practical problems can be solved by using simple simulations. Many are adapted to the computer. Introduction to Probability covers elementary probability by using counting skills and gives students a working knowledge of basic probability. Information From Samples includes surveys and encourages students to ask questions and generate hypotheses about the data.

Oregon Mathematics Concept Paper No. 5

AN INTEGRATED MATHEMATICS CURRICULUM FOR HIGH SCHOOL

PREFACE

The Oregon Mathematics Education Council (OMEC) launched a mathematics curriculum project in May 1984. Nearly 100 enthusiastic, dedicated volunteers, a majority of whom are classroom teachers, worked on the task forces of the project. These groups worked to develop a set of concept papers relating to major issues of reform in mathematics education. Their ideas are shared now as a set of The Oregon Mathematics Concept Papers. They are available as a complete set or as separate papers. You are encouraged to read those papers of special interest and to discuss the ideas with colleagues. Duplicating the papers for such noncommercial purposes is encouraged.

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The OMEC would like to thank Al Elkins of Porter Public Relations, Portland, for his help in editing these papers.

OMEC TASK FORCE
on
AN INTEGRATED MATHEMATICS CURRICULUM FOR HIGH SCHOOL

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INTRODUCTION

The Oregon Mathematics Education Council (OMEC) is a widely representative volunteer group interested in all levels of mathematics education. In 1984 Gary Musser, the president of OMEC, sent out invitations to college and school mathematics educators to a meeting at Oregon State University to discuss the implications of the new three-year mathematics requirement for college entrance, the new two-year requirement for high school graduation, the continuing quandary as to what to do with computers and the lack of emphasis upon problem solving in mathematics instruction. Various task forces were created to examine and propose suggestions to these and other issues.

The Integrated Mathematics Curriculum (originally dubbed "Algebra") Task Force had its first committee meeting in January 1985. Nineteen educators attended, representing middle/junior high school, senior high school, and college mathematics teachers. Several of these educators had a special interest in the role of computers in mathematics education. The purpose of this committee was to make forward-looking recommendations consistent with current developments taking place in mathematics education in the state and nation at large, and with the recommendations being made by other OMEC task forces. It is the hope of the task force that our work will help the Oregon Department of Education and local school districts design mathematics programs which will help our citizens meet their needs in the 21st century.

The unified mathematics approach described in this document reflects three major themes:

- Acceptance of the computer as an ally in improving mathematics instruction
- Sensitivity to students' conceptual difficulties in understanding algebraic concepts
- Consistency with what has already taken place in Oregon

A Unified Mathematics Approach:

The Mathematics Association of America in its famous 1923 report, The Reorganization of Mathematics in Secondary Education, recommended a program of "composite," "correlated," "unified," or "general" courses to be introduced into secondary schools so students might grasp the important interrelationships between arithmetic, algebra, and intuitive geometry in order to gain real insight into mathematical methods. The 1940 report of the Joint Commission of MAA and NCTM, The Place of Mathematics in Secondary Education Fifteenth Yearbook, also advocates a unified approach as its first recommended option. The principles of arranging the topics into courses should be as follows:

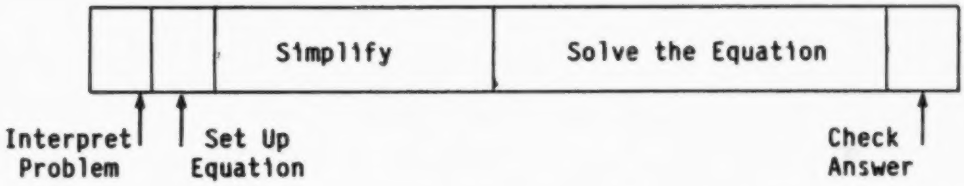
- The sequence of topics should convey a unified mathematical picture. Each topic will contribute toward an ever growing and more significant organization of the basic concepts, principles, skills, facts, relationships, types of appreciations and fields of applications.
- The study should emphasize problem solving and modes of thinking and should not become a mere sequence of formal and relatively abstract drills.

- A new idea or principle should not, as a rule, be introduced prior to the time at which it is needed or may be effectively applied.
- A new topic should not be introduced unless there is sufficient background of prerequisite concepts and skills to permit unhindered concentration upon new elements. (pp. 57-58)

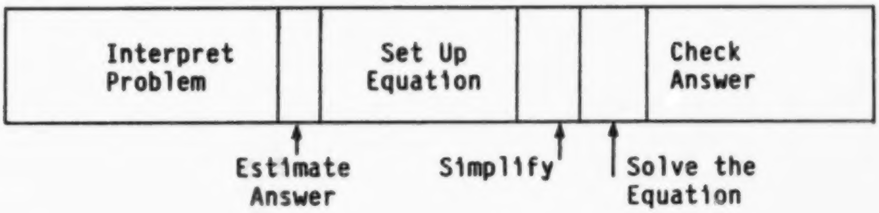
Since the 1923 and 1940 reports, implementation of a unified program has been reasonably successful in many schools in grades seven and eight. The time has come now for this to happen in high school. New York State has an integrated program for those students planning to take the Regents' examination before graduating from high school. Recent Guidelines for the 9-12 Mathematics Curriculum from the Office of the Superintendent of Public Instruction from the State of Washington, recommends three sequences of courses for the integrated study of mathematics for students to select from depending upon their interests and aspirations. The Mathematics Framework for California Public Schools K-12 mandates that concepts from every strand (number, measurement, geometry, pattern and functions, statistics and probability, logic and algebra) be incorporated and interwoven into mathematics course work at each grade level. The task force recommends this approach.

Impact of Computing on Algebra:

A major concern for instruction in algebra is the effect the computer will have on both instruction and curriculum. If algebra is to help students become better problem solvers, then instruction should involve the interpretation of problems, setting up equations, simplifying expressions, solving equations and checking the results. The diagram that immediately follows is an estimate of the portion of instructional time spent on these five important phases in most present-day algebra classes.



Mathematics educators with a special interest in computing, believe that computers can bring about a change in the time allotment as shown in the next diagram.



This shift in time allotment from skills in equation solving and in the simplification of expressions to interpreting problems, setting up equations and checking answers is more consistent with the spirit of mathematics.

With the aid of computers that perform algorithmic manipulations, the conventional order and priority of topics could be inverted. (This concept is expanded upon under the issues section of the paper.) The algebra experience could begin with the notion of relationships between variables, including the methods of determining and expressing such relations, an emphasis strongly advocated by the Mathematical Association of America (MAA) in its 1923 report. (See Computing and Mathematics, pp. 12-13.)

The Conceptual Difficulties in Acquiring Algebraic Concepts based upon Mark Driscoll, Research Within Reach, Secondary School Mathematics

Research conducted in the United States, Britain, and other countries indicates that the mastery of algebraic concepts and principles is much more difficult for students than many had suspected. Results from the recent National Assessment of Educational Progress (NAEP) indicate that "around 40 percent of 17-year-olds with one year of algebra were able to solve linear equations in one unknown. The comparable figure for 17-year-olds with two years of algebra was 60 percent. In both groups, however, the success rate for applying algebraic knowledge to word problems was consistently much lower than 40 percent." The researchers who interpreted the NAEP results noted: "It appears that although additional study in algebra may improve students' 'algebraic skills,' it does little to help them learn to apply those skills to solving problems."

The concepts of variable and equation are critical to algebraic understanding and so misconceptions surrounding these concepts are central to failure in algebra. Some, if not most, of the misconceptions and errors that complicate the learning of algebra are rooted in students' first experiences in algebra and the conceptual frameworks they create to assimilate those experiences. Many students, for example, view the equation as something to do rather than a statement of a relationship and the variables are labels for objects, not numbers. Also, most beginning algebra students bring with them the impression that equations are what they appeared to be in arithmetic, an expression of processes that begin with computations and end with an answer. This impression resists change.

What makes the learning of algebra especially difficult, and so too the teaching of algebra, is the matching of levels of meaning with the levels of learner readiness. Each of the primary algebraic concepts--variable and equation--can have several different meanings, depending upon context. A learner's ability to understand and make use of a particular meaning depends a good part on that learner's cognitive development.

Mathematics Development in Oregon During the Past 25 Years:

For the past 25 years or more, Oregon mathematics educators have been striving to improve math education by writing classroom materials, conducting inservice mathematics workshops, publishing The Oregon Mathematics

Teacher, and participating in the preparation of guides or handbooks for the Oregon Department of Education. Recommendations given in this OMEC document have carefully built upon previous work and experiences.

Most of the recommendations in the 1961 state guide are still applicable today, the following especially so:

> The mathematics content for the various courses should be selected from the point of view of a correlated approach. Such an approach would make it easier for students to see mathematics as an interrelated system of ideas. Arithmetic, algebra, geometry, and trigonometry should be a part of the instruction at many levels rather than one level only. (p. 23)
>
> Teachers, at each level, should organize their instruction around a few of the following points of emphasis: number, operation, measurement, form, and position (theory of space), mathematical symbolism, relationship, proof, and problem solving. (p. 23)

MATH in Oregon Schools:

The curriculum guide, published by the Oregon Department of Education in 1977, was written primarily for teachers (grades K-12) and emphasizes a philosophy of mathematics education with a focus on problem solving. Many examples are given of lessons which have a problem-solving flavor. The document advocated the following strands "to serve as a common base in planning and developing several statewide efforts relating to math education:"

Numeration
Communicating through symbols and models
Problem solving
Computation
Applications
Geometry
Measurement
Estimation and approximation
Probability and statistics

As a goal for mathematics instruction, the task force accepts the position taken by the MAA in its 1923 report:

> The primary purposes of the teaching of mathematics should be to develop those powers of understanding and of analyzing relations of quantity and space which are necessary to an insight into and control over our environment and to the appreciation of the progress of civilization in its various aspects, and to develop those habits of thought and action which will make these powers effective in the life of the individual. (p. 395)

The OMEC task force recommends an integrated mathematics program for both middle school and high school. Therefore, much of what is in the OMEC report, "Oregon Mathematics Concept Paper No. 2: Middle School Mathematics, 6-8," is applicable to the report of this algebra task force. Our report contains three sections:

Section I: Instructional Themes Across the Mathematics Curriculum

Only a statement of the themes will be included. We suggest that the reader refer to the middle school report (pp. 54-58) for details.

Section II: Principles and Generalizations, Skills and Techniques, and Basic Concepts for the First Unified High School Course.

The second and third courses will likely be the subject of future OMEC reports.

Section III: Concerns, Issues, and Points of View

There are concerns unique to the high school mathematics program. These will be mentioned and discussed. Those applicable concerns from the middle school (pp. 72-85) will only be mentioned.

Section I: Instructional Themes Across the Mathematics Curriculum

The following themes cut across all topics and units to be suggested by the principles and generalizations, skills and techniques, and basic concepts in Section II:

Problem solving
Applications of mathematics
Mathematical thinking
Bridging the gap between the concrete and the abstract
Developing concepts
Developing the ability to make estimates
Relational thinking
Reading, writing, and talking about mathematics
Using calculators and computers

Section II: Principles and Generalizations, Skills and Techniques, and Basic Concepts for a First High School Mathematics Course

The content for this first course for high school mathematics should be a continuation and extension of the program recommended in the OMEC paper, "A Mathematics Curriculum for Oregon Middle Schools." In addition, the content should:

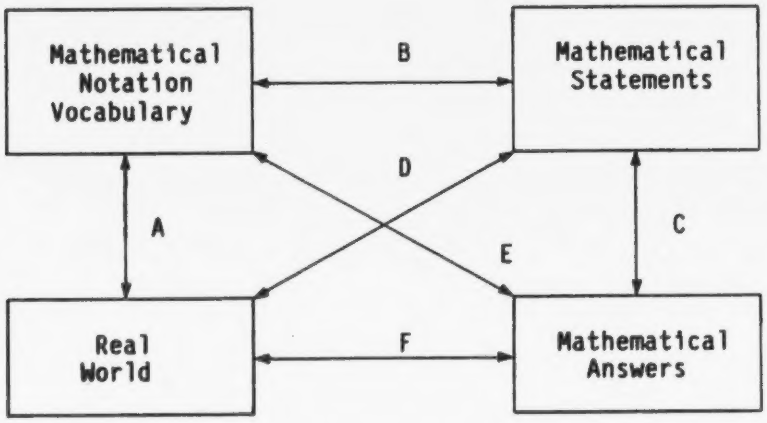
- Emphasize problem solving and other modes of thinking rather than a sequence of formal and relatively abstract drills.
- Include ideas and principles that seem important and useful to students.
- Incorporate the use of the computer as a tool in the classroom.

The listing of content presented here is not of much value unless attention also is given to important learning principles. For a brief discussion of some of these, see the OMEC middle school report under:

Mathematical thinking, pp. 54-55
Bridging the gap between the concrete and abstract, p. 55

Developing concepts, p. 55
Relational thinking, p. 56
Reading, writing, and talking about mathematics, pp. 56-57

The diagram below summarizes the types of instructional activities which should be considered in the learning and teaching of mathematical content.


Mathematical Notation Vocabulary
B
Mathematical Statements
D
A
C
E
Real World
F
Mathematical Answers

Of the types of instructional activities, A-F suggested above, C (mathematical statements to mathematical answers) includes computing. In the past, there seemed to be little time left for activities other than computations. Fortunately, computers and calculators can provide much assistance with C. Possibly we can use some of the time saved for the activities suggested by the double arrows, denoted A, B, D, E, and F. The principles, skills, techniques, and concepts for the first high school mathematics course will be presented as content for the nineteen cells shown in the following diagram.

	Numbers Points Operations	Equations and Inequalities	Relational Thinking	Logical Thinking
	a	b	c	d
I. Arithmetic	Ia	Ib	Ic	Id
II Algebra	IIa	IIb	IIc	IId
III. Geometry and Measurement	IIIa	IIIb	IIIc	IIId
IV. Probability and Statistics	IVa	IVb	IVc	IVd
V. Problem Solving Skills & Strategies Applications	Va	Vb	Vc	☒

Numbers, Points, and Operations

Cell Ia

Arithmetic

Principles and Generalizations

- Field properties and special properties of subtraction and division
- Fundamental theorem of arithmetic
- All whole numbers can be generated by 0, 1 and addition
- Properties of equivalent fractions
- Rules for order of operations
- A series of addition and subtraction of positive and negative numbers can be expressed as the sum of positive and negative numbers.

$a - b = a + (-b)$

$a \mathcal{E} b = a\,(1/b)$ where $b \neq 0$

$a - b = c \ \$ \ a = b + c$

$a \mathcal{E} b = c \ \$ \ a = bc$ where $b \neq 0$

Skills and Techniques

- Compute with rational numbers and integers
- Express a composite number as a product of primes
- Find square and cube roots with a calculator
- Express any decimal in scientific notation
- Order rational numbers when expressed as fractions or decimals
- Express decimals and whole numbers using expanded notation
- Change a fraction to a decimal and vice versa

Basic Concepts

- Integers
- Repeating and non-repeating
- Irrational numbers
- Real numbers
- Base and place value
- Negative exponents
- Scientific notation
- Primes
- Powers
- Ratio
- Percent
- Reciprocal
- Additive inverse or opposite
- Fractions as:
  - Indicated quotient
  - A ratio
- Symbols for indicating order of operation

Cell IIa

Algebra

Principles and Generalizations

Changing subtractions and divisions to equivalent additions and multiplications permit the use of the commutative and associative properties

Properties of a trinomial perfect square

Laws of exponents where exponents are natural numbers

A trinomial perfect square of the form $x^2 + bx + c$ can be factored if the sum of two factors of c is equal to b

Skills and Techniques

Evaluate algebraic expressions

+, -, x, ÷ with monomials and binomials

Combining like terms (application of the distributive property)

Operations with algebraic fractions (denominators generally monomials)

Factoring simple algebraic expressions (common monomial factoring, difference of two squares, $x^2 + bx + c$)

Basic Concepts

Monomial
Binomial
Trinomial
Polynomial
Coefficient
Terms:
- Like
- Unlike

Degree of an algebraic expression
Equivalent forms for algebraic expressions
Prime algebraic expression
Letter as a generalized number
Constant

Numbers, Points, and Operations

Cell IIIa

Geometry and Measurement

Principles and Generalizations

Each point on a number line corresponds to a real number and vice versa

Ratio of the circumference of any circle to its diameter is equal to the constant pi (π)

Measurements are approximations

Answers as a result of calculations with measurements often indicate a misleading degree of accuracy or precision

Skills and Techniques

Add and subtract integers using a number line

Illustrate the distributive property geometrically

Illustrate the product of certain binomials geometrically

Measure segments and angles to the nearest unit

Use direct measuring techniques for determining the perimeter, area and volume of geometric figures (2D and 3D)

Make scale drawings

Describe schemes for representing pi and $\sqrt{2}$ on a number line

Give examples of quantities: whose measures (or numbers) are exact; are approximations

Basic Concepts

Number line

Absolute value as distance

Points as numbers on a number line

Points as an ordered pair of numbers in a coordinate plane

Union and intersection of lines, line segments, rays

Precision

Significant digits

Tolerance

Triangular and rectangular numbers (Figurative numbers)

Union and intersection of:
- Lines
- Line segments
- Angles
- Rays

Numbers, Points, and Operations

Cell IVa

Probability and
Statistics

Principles and Generalizations

Advantages and disadvantages of using mean, median, and mode in measures of central tendency

Central tendency is a number which can be used as a suitable representative for a set of numbers

The probability of an event can never be less than zero or greater than one

Skills and Techniques

Determine probabilities or odds

Determine mean, median and mode from collected data

Determine suitable scales to be used when graphing data

Collect or organize data which can be used to answer some question or solve a problem

Tabulate data and make a frequency distribution

Make line, bar and circle graphs

Draw justifiable inferences from data

Give examples of problem situations where sampling procedures can provide reasonable solutions

Basic Concepts

Probability expressed as a decimal or fraction

Odds

Expected outcome

Sample

Representative sample

Frequency

Central tendency:

- Arithmetic mean
- Median
- Mode

Numbers, Points, and Operations

Cell Va

Problem Solving
Skills and Strategies
Applications

Describe plausible situations in which operations with signed numbers (including rational numbers) have meaning
Organize statements of basic arithmetic facts and look for patterns which extend to the integers
Organize powers expressed in exponential form and look for possible meanings that might be given to zero and negative exponents
Vary numeration problems to a place value system other than ten: e.g., $4(10^2) + 9(10^1) + 5(10^0) + 3(10^{-1}) + 8(10^{-2}) = 495.38$
What does $4(5^2) + 2(5^1) + 3(5^0) + 3(5^{-1}) + 1(5^{-2})$ equal?
Check correctness of operations with algebraic expressions by substituting numbers for letters
Determine possible coordinates for a fourth point of a rectangle when the other three points are plotted and their integral coordinates are known
Solve a variety of consumer mathematics and personal finance problems involving fractions, decimals and percent
Plan and make a survey as a method of coming to some conclusion or substantiating some conjecture
Give examples of ways probability and statistics can be used in the solution of realistic problems: e.g., insurance, gambling, investment and decision making

Equations and Inequalities
(Mathematical Sentences)

Cell Ib

Arithmetic

Principles and Generalizations

Properties of proportion

Skills and Techniques

Find the fourth term of a proportion when the other three are given

Write other proportions from the terms of a given proportion: e.g., $\frac{3}{4} = \frac{9}{12} \$ \frac{12}{4} = \frac{9}{3}$

Basic Concepts

True and false statements

Statements involving <, >, =

Equations and Inequalities
(Mathematical Sentences)

Cell IIb

Algebra

Principles and Generalizations

Properties of equivalent equations
Properties of simultaneous equations (1st degree two variables)
In $ab = 0$ either a and/or b must equal zero

Skills and Techniques

Evaluate formulas
Use guess and check procedures for solving simple 1st and 2nd degree equations (one variable)
Write a more complex equivalent equation for a simple equation: e.g., $x = 5 \rightarrow 3x - 2 = 13$
Write a pair of simultaneous equations equivalent to a given pair
Use properties of equivalent equations for solving simultaneous equations
Use substitution method for solving simultaneous equations
Use guess and check procedures for solving inequalities
Invent simple equations with one, no, or more solutions
Use factoring for solving factorable quadratics
Use quadratic formula for solving quadratics
Solve for any variable in formula or equations of the form $ax + by = c$

Basic Concepts

Unknown
Variable:
- Dependent
- Independent

Equivalent equations
Equivalent pairs of simultaneous equations

Equations and Inequalities
(Mathematical Sentences)

Cell IIIb

Geometry and
Measurement

Principles and Generalizations

- Triangle inequality
- Angle sum property of triangles
- Side and angle properties of similar triangles
- Formulas for area of rectangle, triangle, parallelogram and trapezoid
- Formulas for volume of rectangular solids, cones, cylinders, pyramids and spheres
- Arc of a circle has the same measure as its central angle
- The inscribed angle of a circle equals 1/2 its intercepted arc
- The sum of the interior angles of a polygon equals 180(n-2)
- The maximum number of lines determined by n points = n(n-1)/2

Skills and Techniques

- Find the solution of simultaneous equations graphically
- Find missing measures of a right triangle using the principles of similarity or the Pythagorean relation
- Use formulas for finding perimeter, area, volume and lateral area
- Find the measure of an interior angle of a regular polygon
- Find the slope of a line graphically
- Solve quadratic equations graphically
- Sketch the graph of a quadratic equation that has no solution (real number solutions)
- Find the solution of a line or equation graphically

Basic Concepts

- Arc
- Angle degree
- Arc degree
- Central angle
- Inscribed angle
- Regular polygon
- Base

Equations and Inequalities
(Mathematical Sentences)

Cell IVb

Probability and
Statistics

Principles and Generalizations	Skills and Techniques
The number of two-member committees possible with n persons = $n(n-1)/2$	Use inequalities to describe domain and range of statistical data

Cell Vb
Problem Solving
Skills and Strategies
Applications

Translate story problems (which are meaningful to students) and geometric situations into algebraic sentences
Invent situations which can be described by algebraic sentences
Check an equation by determining whether or not the answer is reasonable (a reality check)

Relational Thinking

Cell Ic

Arithmetic

Principles and Generalizations

State how a third number in an arithmetic statement (including rational numbers) changes as the other two numbers change in some well defined way

State how two of the three numbers in an arithmetic statement must change if the third number does not change: e.g.,

$32 \div 8 = 4$
$32 \div 4 = 8$
$32 \div 2 = 16$
$32 \div 1 = 32$
$32 \div 1/2 = 64$
$32 \div 1/4 = 128$,
etc.

Skills and Techniques

List in a systematic way integers whose differences are always the same integer (also for sums)

List in a systematic way rational numbers whose quotients are always the same number (also for products)

Basic Concepts

Decrease or increase without bound

Relational Thinking

Cell IIc

Algebra

Principles and Generalizations

- Properties of a table of values for $ax + by = c$
- Properties of a table of values for $y = mx + b$
- Direct variation $y/x = K$ [K is not 0] or $y = kx$
- Inverse variation $xy = K$ [K is not 0]
- Explain how percent problems might be considered as examples of direct or inverse variations

Skills and Techniques

- Make a table of values for two variable first degree equations and describe how one variable changes as the other changes
- Make a table of values for an equation of the form $y = ax^2 + bx + c$ and describe how y changes as x changes
- Predict additional ordered pairs for a table of values
- Write the rule or equation for a table which displays a simple relationship (using a guess and check procedure)

Basic Concepts

- A constant treated as a variable
- Vary in the same sense or opposite sense
- Direct variation
- Inverse variation
- Independent variable
- Dependent variable

Relational Thinking

Cell IIIc

Geometry and
Measurement

Principles and Generalizations

Explain why equations of the form $ax + by = c$ and $y = mx + b$ are called linear equations
The slope between any two points on a line is always the same
The two-point equations for a straight line
Two lines with the same slope are parallel
The graph of inversely related variables does not intersect either the x or y-axis
The graph of two directly related variables is a line that passes through the origin
Pythagorean relation
Congruent triangle properties
Parallel lines and transversal relationships
Side, angle, and diagonal properties of a kite; of a rhombus

Skills and Techniques

Make graphs from tables of values and vice versa
Make tables of values for two-variable formulas and graphs
Determine the slope and y-intercept of a line from its graph or table of values
Predict the equation for a line from its graph, its table of values, check results
Given the coordinates of two points on a line, determine its equation
Construct one triangle congruent to another using SSS, SAS, or ASA
Make a table of tangent ratios for 10, 20, 30,...80 degree angles
Describe how the tangent varies with respect to the angle size
Use compass to construct:

- Line perpendicular to another line through a point on the line, off the line...
- Bisector of a line segment and an angle
- Line parallel to a line through a point not on the line
- Angle congruent to another angle

Use tangent ratio for measuring inaccessible distances

Basic Concepts

y-intercept
Slope: positive, negative, zero
Congruent angles; segments
Perpendicular lines
Angles:

- Adjacent
- Vertical
- Supplementary
- Complementary

Tangent ratio

Relational Thinking

Cell IVc

Probability and Statistics

Principles and Generalizations

- A statistical generalization is not destroyed because of a few exceptions
- A statistical generalization applies to the population generally, but not necessarily to individuals in the population
- The arithmetic mean has the disadvantage of being greatly affected by the extreme measure
- The mode always reflects the most popular item or class
- The median is the only measure of central tendency that gives equal weight to all items

Skills and Techniques

- Collect, organize, and graph statistical data
- Estimate the line of best fit
- Make a frequency distribution and histogram
- Determine a given percentile or quartile
- Conduct a probability experiment and compare cumulative relative frequency with theoretical probability
- Draw conclusions which can be supported by graphs and charts taken from current periodicals

Basic Concepts

- Percentile
- Quartile
- Cumulative frequency
- Relative frequency
- Line of best fit

Cell Vc

Problem Solving
Skills and Strategies
Applications

Discover relationships by collecting data, organizing data, and looking for patterns:
- Operations with positive and negative numbers
- Pythagorean relation
- Number of diagonals from a single vertex of a polygon
- Total numbering diagonals of a polygon
- Sum of interior angles of a polygon
- Hookes Law
- Law of the lever
- Laws of exponents (extensions to zero and negative exponents)

Use surveying devices for finding inaccessible distances (tangent ratio)
Invent equations which have a given solution, including complicated ones
Determine probabilities of an event experimentally
Select problem settings in which direct or inverse variation applies
Program a calculator or computer to solve realistic problems

Logical Thinking
(Deductive Thinking and Proof)

Cell Id
Arithmetic

Provide rationale for:

- The difference between two numbers can be greater than either or both numbers: e.g., $5 - (-2) = 7$; $-7 - (-2) = -5$
- The product of two negative numbers is a positive number
- The sum (or total) of several numbers can be less than at least one of the addends: e.g., $(16 + -3 + -2 + -6 = 5)$
- a/b meaning _a_ divided by _b_
- Procedure for changing a fraction to a decimal, a decimal to a fraction
- How to order and compare numbers

Cell IId
Algebra

Provide a rationale for $x^0 = 1$; $x^{-1} = 1/x$
Substitute numbers to check results of an algebraic operation
Check solutions to equations
Informal use of equivalent equation properties for the steps in the equation solving process
Show why division by zero cannot be allowed
Show why there is need for a rule for order of operations when a series of operations are to be performed
Show why the sum of two even numbers would always be even (and similar examples)

Logical Thinking

(Deductive Thinking and Proof)

Cell IIId
Geometry and Measurement

Demonstrate why an equation of the form ax + by = c is called a linear equation
Show why the sum of exterior angles on convex polygon equals 360 degrees
Show why the sum of the interior angles of a polygon with n sides = 180(n-2)
Show why an exterior angle of a triangle is equal to the sum of the opposite interior angles
A counter example disproves a mathematical generalization
Prove informally the Pythagorean relation
Provide an informal proof of the compass constructions using congruence and/or rhombus properties
Provide an informal proof for the sum of the exterior angles of a polygon being 360 degrees

Cell IVd
Probability and Statistics

Make a tree diagram or chart to show that the theoretical probability of getting:
- Two heads when two coins are tossed (1/4)
- The sum of seven when a pair of dice is tossed (1/6)

Predicting probabilities for obtaining sums from a pair of dice shaped as tetrahedrons, octahedrons, etc.
Make inferences from probability statistics for a given situation (smoking, lottery, gambling)
Identify misleading inferences from statistics
Support conclusions using statistics

Section III: Concerns, Issues, and Points of View

History of a Unified Approach:

1. A unified (or integrated) mathematics curriculum has been recommended by influential national committees in 1923 and 1940. Why haven't these recommendations resulted in the publication of suitable textbooks, and in what way are conditions more promising now?

   One question that has been with us for a long time is, "How should we organize the mathematics we teach?" Probably the most influential report for the establishment of the program of a year of algebra, followed by a year of geometry, was the National Education Association 1899 report on College Entrance Requirements. (Bidwell, James K., and Robert G. Cason, ***Readings in the History of Mathematics Education***, NCTM, 1970). There were thirteen members on the committee, only one of which was a mathematician. This decision influenced the publication of high school mathematics textbooks for a 20-year period when the high school population increased eight fold. This entrenchment of the idea of two parallel courses has not been broken.

   Many reforms in education, psychology of learning, and sociology have taken place since then, along with the social upheavals of wars, a great depression, and the civil liberties movement of the late 1960s and early 1970s. These strong influences have led to changes in the content of textbooks for both the algebra and geometry courses. However, the 1899 recommendations for separate courses for algebra and geometry are still the dominant organizational pattern in the United States, even though committees of mathemeticians and mathematics educators have continued to recommend unified or integrative approaches. Appropriate textbooks have been published, one as early as 1906, others in the late 1920s and early 1930s, and a series of texts by a company in the 1950s. At the present time (1986), at least three textbook companies have the series to satisfy the demand in states where the departments of education recommend such programs. The availability of such texts is better today than any time since the turn of the century. One hopeful sign!

   During much of this century, the prevailing psychologies of learning, connectionism and behaviorial approaches did not necessarily require a unified approach to mathematics instructors. The case for a unified curriculum is based upon such process goals as meaning, understanding, creative thinking, problem solving, and critical thinking. Mathematics educators feel these goals are more compatible with the more recent gestalt views of learning. Today, the process goals and gestalt views toward learning are popular with a large percent of the educators. A second hopeful sign!

   Probably, the most significant development that favors a unified approach is the advent of computers and calculators. These electronic devices relieve humans of tedious and routine computations, but require more time to be spent on problem solving and searching for relationships within mathematics itself and with the real world. Many computer and mathematics educators suggest an inverted or top-down approach (see issue 3) where instruction begins with a problem and the necessary details are dealt with as needed. Such an approach is at the center of a unified approach to mathematics instruction. (See NCTM, Computing and Mathematics; NCTM, A History of Mathematics Education in the U.S. and Canada, 1970; Bidwell, James K., and Robert G. Cason, Readings in the History of Mathematics Education, NCTM, 1970.)

Computers and Mathematics:

2. What is the role of the computer in the teaching of mathematics?

There are microcomputer programs available today for the classroom that can:

> . . . perform almost all the symbolic manipulation necessary in academic mathematics classes through first year calculus. (Fey and Heid, 1985)

Algebra is now taught from an equation solving perspective. Students are presented with a variety of ways to manipulate equations to produce solutions. Time spent concentrating on concepts and real world situations occurs in the second or third year of algebraic instruction. Before the advent of the microcomputer, this was a natural sequence because it was necessary for students to be able to obtain equation solutions.

If computers are capable of solving equations, then how much instruction should be devoted to students mimicking the equation solving capabilities of microcomputer programs, and how much to learning algebra and deciding when to use a computer, and when not to? The use of the computer makes it possible to allocate more time of a first year algebra program to concepts and applications. Time can be spent on organizing information, understanding relationships, translating situations into algebraic equations, and gaining an intuitive grasp of equations and their solutions.

Calculators and computers have had an immense impact upon our society, yet these changes have only been partially realized within education. In mathematics, it is forcing an analysis of the curriculum. The goals remain the same. However, the overall curriculum is changing. Just as technology is forcing this change, it is also providing new avenues to achieve the goals.

The questions to be posed in teaching the concepts in mathematics are:

Does technology change:
- The place the concept is introduced?
- The skills needed to apply the concept?
- The level of understanding that can be reached?
- An awareness of the concept's place in the students' environment?
- The manner in which a topic is taught?

For example, it is still necessary to teach positive and negative integer concepts; however, students need not do extensive manual calculations to solve an equation like $213.45\ x = (48.4x-909.13)/1.4$. However, they do need to understand the underlying concepts. The use of the calculator has changed the need for computations with logarithms to a need to be able to use the rules of exponents.

Technology has also affected the level of understanding a student needs. In an age where a machine can make a billion calculations a second, students can no longer afford to be just calculators. They must be able to select the data to be processed, organize it, determine the manner in which it is processed, interpret the results, and present the material.

Computers provide a nonthreatening, interactive, exploratory environment. This allows for topics to be investigated in a nonjudgmental and creative atmosphere. In this world, a teacher's role shifts from being a provider of information to a facilitator of learning.

In this technological society, students must be able to select the best method for solving a problem. Tools extend and enhance mathematical abilities, not supplant them. Students must be equally competent whether using estimation, pencil and paper, mental arithmetic, or calculator and computer skills.

Therefore, within the curriculum there needs to be a shift from drill and practice to applications, from tedious calculations to problem solving, from isolated topics to integrated activities.

Inverted Curriculum:

3. Many computer and mathematics educators advocate an inverted curriculum. What does this mean?

Fey (1984) and others have suggested that one of the major impacts of the application of computers in mathematics education will be to invert the traditional organization of the curriculum. Through the computer's graphic and symbolic manipulation capabilities first-year students can deal with advanced algebraic problems and concepts, without worrying about the details of solving equations. This means that the abilities students now need are those of (Fey and Good, 1985):

a. Recognition and mathematization of relations through the use of motivating and real world situations.

b. Answering questions about functions by using increasingly sophisticated techniques such as guess and check by hand, then computer-generated tables of values and graphs, then exact solutions by computer programs.

c. Formal organization and verification summarizing principles and methods discovered in exploratory work.

The gist of the argument is that computers and software tools will give students the power to see and explore much larger portions of the mathematical forest than the tree-by-tree curricular approach that is common today.

4. What is an example of a math lesson using this unified math approach that integrates the use of the computer as a tool?

The following unit plan is an elaboration of the example of the inverted curriculum. It offers a model of how this concept might be applied within the context of the unified mathematics curriculum proposed by this report. Several key ideas need to be mentioned at the outset.

a. The organization of the unit is from general themes and problems, explored through the use of the Algebra Tool Box, to the development of specific concepts and skills (see issue 5).

b. There is an integration of content that emphasizes simultaneous study of several of the content strands proposed in the report.

c. There is an attempt to draw on problems that will encourage application and transfer of concepts and skills across curricular lines.

Mathematical Modeling First Degree Polynomial Relations

PROBLEM EXPLORATION

Objectives:

a. Become acquainted with using algebra as a language to describe relationships.

b. Compare and contrast English language, algebraic, and graphic descriptions of relationships.

Prerequisites:

a. Ability to read and interpret graphs and formulas.

b. Ability to use the Algebra Tool Box to:

   1) Enter and graph data.

   2) Write summaries of problems.

   3) Enter and manipulate formulas.

   4) Do automatic regression analysis.

Activities:

a. Small groups of students assigned to collect and organize data related to several of the type of problems suggested by the following examples. A large pool of such problems should be available. There should be overlap of problems from one group to another to allow for comparison of analyses. The problems should all require active collection of data on the part of students (e.g., experimentation, library research, or survey).

Sample Problems:

1) Trace and describe the value of a new car from new sticker price through successive years of blue book value.

2) Describe changes in the world record in the mile run since 1900.

3) Experiment with and describe the relationship between time, rate, and distance.

b. Use the Algebra Tool Box to:

1) Organize and present the data collected in table and graph form.

2) Determine an equation that describes the data using the regression function of the tool box.

3) Test the equation by generating a new table of values and graphing the equation along with the data.

4) Prepare a written description of the relation.

c. Give an oral presentation to the class describing each of the problems studied.

5. What kind of computer software will be available for use in teaching mathematics?

Currently most of the components of this proposed piece of software are available for the classroom. Here is a sample of what students may have available in the near future.

ALGEBRA TOOL BOX

Calculator:
    Performs Arithmetic
    Statistical Analysis

Data Analyzer:
    Pattern Seeker
    Function Generator
    Regression Analyzer

Equation Solver:
    Symbol Manipulator

Grapher:
    For visualizing equations and relations
    Can rotate, reflect, translate and stretch three dimensional projections

200

Programming Language:
    Simple language or language subset for designing more math tools

Slate:
    Place for listing and sorting information
    Variable identification and relationships
    Boundary conditions for problem and graph

Tutor:
    May be turned on or off
    Can help step by step or take over and solve
    Can generate a similar problem to solve

Word Processor:
    For writing explanations of the particular problem solving approach chosen, and interpretation of the results

Probability and Statistics:

6. What is the role of probability and statistics in high school mathematics?

Students need to be exposed to the kind of mathematical thinking which not only applies to simple problems of everyday life, but which also provides the basis for modern scientific thought. The concepts taught in algebra lend themselves to applications. Unfortunately, few algebra texts actually include these applications as part of the course of study. If we consider one of the major goals of algebra as being able to take a real world problem, write it in mathematical symbols, and then solve it, the study of probability theory and statistical inference are natural candidates to help facilitate this goal.

Looking at the increased amount of data people are expected to process in our society, should provide convincing evidence that work with statistics and probability is essential for all students. The grade levels where exposure to these topics is most lacking is 9-12. Approximately 75 percent of all high school students take the typical Algebra I class before they graduate. For this reason alone, the greatest number of students can be reached if we expose them to these topics in their first high school mathematics course. The question becomes, "How?"

Teach the first year unified math course for high school through applications which incorporate elements of probability and statistics. In 1976, NCTM published such a text which was supported by a National Science Foundation grant, directed by Zalman Usiskin, University of Chicago. To get an idea of how this was done, here are some examples where topics were incorporated.

Chapter 1:	Some Uses of Numbers Bar Graphs TV Ratings and Sampling Relative Frequencies -- Another Use of Ratios
Chapter 3:	Addition and Subtraction A Statistic--The Mean A Statistic--The Mean Absolute Deviation

Chapter 5: Models for Division
Probability of Outcomes
Probability of Events
Chapter 9: Slopes and Lines
Types of Graphs--Line of Best Fit could be here
Chapter 10: Powering
The Permutation Model of Powering
The Growth Model of Powering
Chapter 13: Sets and Events
The Abbreviation P(E) and N(E)
Expected Frequencies
Unusual Events
Independent Events
The So-Called "Law of Averages"
Chapter 16: Functions
Probability Functions
Frequency Distributions
The Chi-Square Statistic

Second-and Third-Year Unified Math Courses:

7. What can be said at this time about the nature of the second and third unified mathematics courses?

The principles and generalizations, skills and techniques, and basic concepts for the second and third unified courses are to be presented in nineteen cells similar to the first course. The first course is built upon the OMEC middle school program. Therefore, the second and third courses will be a continuation and an extension of the first course. All three courses should include the content specified in the OMEC document "Recommended High School Mathematics for College Admission." The first two courses stress content important for all high school graduates. The third course will be directed more toward those students who plan to continue their education in college or university. The content, then, should include topics which provide an intuitive introduction to basic ideas from probability, statistics, matrix theory, game theory, and numerical analysis, as well as from calculus.

Concrete Materials:

8. What role does the use of concrete materials have in the high school math curriculum?

a. Use physical objects to build number patterns.
b. Ideas can be developed from patterns.
c. Use concrete materials to clarify ideas.

Mathematics teaching is often seen as an initiation into rules and procedures which, though very powerful (and, therefore, attractive to teachers) are often seen by students as meaningless. It follows that students' methods and their levels of understanding need to be taken far more into account. (Dietmar Küchmann, "Algebra," Chapter 8. Children's Understanding of Mathematics. KorHart: London, England.)

Ideally, mathematical concepts and principles should be formed under conditions where thought is necessary, as opposed to acquisition solely from routine or repetition. It is the teacher's role to set up a learning environment where that occurs. If mathematical concepts and principles are first demonstrated with concrete models and later formalized, when appropriate, then students who get lost have a concrete model to return to for clarification of the concept or principle.

Students should have much experience in generalizing to help them see algebraic concepts as a way of representing patterns and relationships which occur in the real world. Physical objects and pictorial representations can be used to represent relationships or number patterns. Students should then be given time to explore those relationships or patterns and develop ideas from them. After this has occurred, teachers need to guide students to the symbolic representation of the nth case or generalization of the concept being explored.

When models are used to teach a concept, teachers need to be aware of any limitations the model might have. For example, if the number line is used to illustrate addition concepts, can it be extended to also illustrate subtraction, multiplication, and division or will the model have to be changed?

Another way to make equations, relationships, and expressions more concrete is to establish an arithmetic framework. Students can be encouraged to check the truth of statements by substituting numbers for the variables, and numerical models can be used by students to help them avoid frequent errors made in symbolic manipulation.

A guess and check procedure can be used to think about reasonableness of answers and to show relationships in problem situations which can lead to the translation of problems into mathematical notations.

For additional comments on this question see "Bridging the Gap Between the Concrete and Abstract" and "Developing Concepts" in the OMEC report, "Oregon Mathematics Concept Paper No. 2: Middle School Mathematics, 6-8," (p. 55).

Cooperative Learning:

9. What is the role of cooperative learning in the secondary mathematics classroom?

The use of groups in the classroom is effective whether in elementary, secondary, college, or technical training. The social development of the student is not the major concern here; it is the student's ability to better learn and understand mathematics. This can be achieved more efficiently and effectively with cooperative learning.

A recent study by Dorothy Cox and Carl Berger ("The Importance of Group Size in the Use of Problem-Solving Skills on a Microcomputer," J. Ed. Computing Research Volume 1(4), 1985) demonstrated that students using computers solve more problems, in less time, and demonstrate more understanding when they work in groups of two, three, or four. Working individually does not offer the variety and additional understanding that communication with others provides. A group of five or more is a party which usually splinters and produces some students who do not profit from the process.

The key ingredients each group must include are trust, communication, and cooperation. It is the teacher's job to use activities that encourage and enhance these attributes. (Also see OMEC, "Oregon Mathematics Concept Paper No. 2: Middle School Mathematics, 6-8," p. 83.)

Evaluation and Standardized Tests:

10. How will the scores on standardized tests taken by students in Oregon in a unified high school mathematics program compare with students in the present program?

Students who would take the unified courses should do as well or better on such standardized tests as students in the present high school program. Standardized tests such as the California Tests of Basic Skills (CTBS) are used by many school districts to compare their students with those in other districts. The specific mathematics objectives of these tests are also objectives of the unified programs OMEC advocates. Of course, the OMEC program includes many objectives which are not tested by CTBS.

Some time ago the College Entrance Examination Board (CEEB) made a shift in policy from examining achievement for specific mathematics courses to general tests predictive of college success--tests such as the Scholastic Aptitude Tests (SAT). CEEB is more interested in testing students' ability to use the knowledge they have acquired in unfamiliar situations, solving problems, and applying critical thinking. OMEC continues to support and encourage the development of these skills in the mathematics classroom. We feel that a unified course will accomplish these goals better than the present scheme of separate courses for algebra and geometry. (For additional discussion of evaluation and standardized tests see "Oregon Mathematics Concept Paper No. 2: Middle School Mathematics, 6-8," pp. 81-83.)

Vertical Acceleration:

11. To what extent and how should vertical acceleration be used for high-achieving and mathematically talented students?

A fairly small percent of students are intellectually ready for algebra in the eighth grade. This percent should not change significantly once this curriculum is implemented. Since this approach is based on the cognitive development of children, relevant applications to the real world, integration of computers and a unified approach students can acquire the needed enrichment within the regular classroom framework without vertical acceleration.

An unpublished OMEC paper, "Acceleration/Enrichment," is available from Don Fineran.

Staff Development:

12. What are the recommendations for staff development for secondary math teachers?

Teachers and curriculum coordinators need inservice in two content areas: 1) computer use in the classroom; and 2) design and use of concrete materials. Additional staff development time should be concerned with recent results in cognitive science and its impact on education. Training should be designed incorporating research results in the area of staff development.

Just a few guidelines for inservices include:

- District, building and teacher support are all necessary for productive, ongoing inservice workshops.
- One-time workshops are not effective.
- Training only one person at a school and expecting them to implement all the changes and new ideas is not effective.
- Inservices should include groups of teachers from a site with ongoing training and support.

In order for teachers to effectively use computers in the classroom, they need to understand the capabilities and limitations of the computer hardware design and the software available for use in education. This does not mean they need to become computer scientists or even know how to program a computer. The focus should be on HOW the computer can be used to teach appropriate mathematics in an efficient and effective manner.

How to design, select, and use materials at the secondary level is important, yet it is not required in preservice training. The goal is to have students thinking abstractly, but in order to attain that goal more students need to begin at the concrete level and develop the foundation for the concepts first.

Recent information concerning the importance of memory and the ability to solve problems, different learning modalities, stages of cognitive development and other educational issues can make teaching easier and more interesting for both the teacher and the student.

13. What mathematics courses should be included in a high school program?

A high school should have a mathematics course appropriate for any student who wishes or is required to study mathematics. The course should be challenging enough to be interesting, yet realistic enough for the student to be successful. This might imply that in addition to the three unified courses, analysis, and calculus, there should be four alternative courses.

Unified Math Courses	Alternatives
UM 1	ALT 1 - equivalent to second middle school math
UM 2	ALT 2 - a third middle school math course pre UM 1
UM 3	ALT 3 - for students undecided about college prep
Analysis	ALT 4 - follows UM 3, modern topics
Calculus	

ALT 4 includes content from probability, statistics, matrix theory, game theory, and/or number theory. It is an alternative for those students who choose not to take calculus in high school.

Some sequences of courses a student might follow are shown below:

	I	II	III	IV	V	VI	VII	VIII	IX
Gr 9	UM 1	UM 1	UM 1	UM 1	ALT 2	ALT 1	UM 2	UM 2	UM 2
Gr 10	UM 2	UM 2	UM 2	UM 2	UM 1	ALT 2	UM 3	UM 3	UM 3
GR 11	UM 3	UM 3		ALT 3	UM 2	UM 1	Analysis	ALT 4	Analysis
GR 12	Analysis	ALT 4	ALT 3	UM 3	ALT 3	UM 2	Calculus	Analysis	ALT 4

The responsibility of the placement of students in these courses belongs to the teachers and counselors for both the middle schools and high school, as well as the students themselves.

Oregon Mathematics Concept Paper No. 6

OREGON GRADUATION REQUIREMENTS: MATHEMATICS

PREFACE

The Oregon Mathematics Education Council (OMEC) launched a mathematics curriculum project in May 1984. Nearly 100 enthusiastic, dedicated volunteers, a majority of whom are classroom teachers, worked on the task forces of the project. These groups worked to develop a set of concept papers relating to major issues of reform in mathematics education. Their ideas are shared now as a set of The Oregon Mathematics Concept Papers. They are available as a complete set or as separate papers. You are encouraged to read those papers of special interest and to discuss the ideas with colleagues. Duplicating the papers for such noncommercial purposes is encouraged.

OMEC is a widely representative advisory group, representing teachers at all school levels, counselors, administrators, school boards, business, labor, industry, and parents. The Oregon Mathematics Concept Papers are being disseminated by the Oregon Mathematics Project, working with a number of mathematics organizations, and many individuals. The organizations are:

Oregon Mathematics Education Council
Oregon Council of Teachers of Mathematics
Oregon Math Leaders
Teachers of Teachers of Mathematics
Community College Math Chairers
Honors Institutes

For additional copies of The Oregon Mathematics Concept Papers, or information regarding speakers and/or discussion leaders, contact:

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The Oregon Department of Education supports, facilitates and coordinates ongoing efforts among these organizations, including the OMEC Curriculum Project. Don Fineran serves as Department liaison to all of the above organizations.

The OMEC would like to thank Al Elkins of Porter Public Relations, Portland, for his help in editing these papers.

OMEC TASK FORCE
on
OREGON GRADUATION REQUIREMENTS: MATHEMATICS

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FOREWORD

With the adoption of the two-year graduation requirement in mathematics, the State Board of Education has created an opportunity for the promotion of a general education of high quality in mathematics for all students. In setting this requirement, the State Board of Education is fulfilling its responsibility to set general standards for the system of education in Oregon. Within these standards, however, there exists the flexibility for local districts to establish educational programs that are responsive to local need.

The new graduation requirement provides the potential for greatly improving mathematics education in Oregon. That potential can be reached through well informed local and state leadership. To assist responsible implementation of minimum standards, the Oregon Mathematics Education Council has assigned a committee to research and identify minimum standards and suggest ways to implement these standards. For schools not presently providing a program that meets the minimum standards, the committee intends to provide direction to assist schools in meeting the standards. For those schools that are presently providing such a program, the committee suggests alternatives that can enhance their existing programs. But for all schools, it is the intent of the committee to establish some basic guidelines for meaningful mathematics programs.

ESSENTIALS OF A HIGH SCHOOL MATHEMATICS PROGRAM

Mathematics is both a tool and a means to communicate. The vast array of quantitative data and information confronted daily makes mathematics a fundamental tool for exercising responsible citizenship. It is an essential means for becoming a contributing member of society. It provides the individual with a necessary capability for satisfying personal needs and interests. It is a language and structure for communicating ideas to others and to one's self.

The following student goals identify the rich and diverse applications of mathematics that each individual will need as an adult.

1. To recognize and use the mathematics necessary for one's own consumer affairs, selecting and using goods and services wisely--choosing and buying personal items and products for a home, for a family or for a small business.

2. To recognize and use the mathematics necessary for other quantitative aspects of one's daily life--mathematics related to cooking, sewing, gardening, minor home or auto repair, personal projects, money management and recreation.

3. To possess requisite mathematics to enter a job situation--appropriate background, learning skills and attitudes so that additional job-related applications of mathematics can be learned with relative ease.

4. To use logical reasoning and problem-solving skills in reaching decisions, weighing evidence, using data and interpreting information to arrive at sound predictions, decisions and conclusions.

Achievement of these student goals requires understanding and reasonable competence in arithmetic computation including mental, paper/pencil and calculator components. This understanding and competence must be supported by:

1. Mental computation, both precise and approximate, with awareness of reasonableness of answers.

2. Understanding of, and ability to use, simple algebraic formulas.

3. Ability to interpret and analyze graphs and statistical information.

4. Knowledge of geometric forms and spatial relations.

5. Understanding of and ability to use measuring instruments.

6. Problem-solving and logical thinking abilities, ranging from skill with simple word problems to more complicated analysis of situations requiring an awareness of productive reasoning techniques.

7. An awareness and appreciation of the capabilities and limitations of devices for carrying out mathematical transactions such as the calculator, computer, electronic banking machines or food market price readers.

In addition to college preparatory course work, high schools need to offer strong alternative instruction for students who lack the interest or ability for this traditional program. Because of the wide range of student abilities, computational skill development has in the past been emphasized to the exclusion of activities involving the application of mathematics to real situations. As a consequence of this approach, many students perceive little relationship between classroom activities and the skills necessary in the nonclassroom world. For too many students, mathematics never becomes an operational tool in their lives.

The fact that a second unit of mathematics is being required for graduation provides a strong incentive to develop new opportunities and sequences at the high school level. If carefully articulated with seventh and eighth grade courses, these courses should help assure a well-integrated and strong mathematical preparation--broad in nature through grade nine and more specialized thereafter.

Coursework which is clearly remedial (below the middle school level) should be confined to intervention programs and not offered as an alternative for the first or second year of high school math credit.

Some objectives, general approaches and methodology for a general mathematics course is described in a subsequent section. Some suitable alternatives for the second unit required for graduation are also discussed.

OPTIONS FOR THE FIRST YEAR

The initial course in high school mathematics is varied depending on the student's entering skill level and ability. The traditional course of Algebra I or above remains a viable option and will not be elaborated upon in this paper. At the general math/pre-algebra level, students seem to fall into three levels:

1. Students requiring special education. Students of this type receive instruction from a resource specialist. The instruction is generally individualized. If students make sufficient growth, they are mainstreamed into the general high school program. This course normally does not count as one of the two required high school math credits.

2. Students with behavioral and/or basic mathematics problems who may experience difficulty with group instruction, and often have low reading and/or listening skills. This group has limited retention and hence may lack memorization of multiplication or addition facts. These students need special classes with small enrollments (10 to 15 students) or individualized instruction. This course is typically labeled basic or general math.

3. Students more able to process group instruction with skill/concept level more advanced than that of students in groups 1 and 2, but who lack prerequisites for algebra either in terms of content development or capability for abstract thought. This group typically comprises the ninth grade general math or pre-algebra course. Class size should be limited to enrollment of not more than 20 students.

One of the instructional goals for all of these students must be to improve their confidence in their own mathematical abilities. Lessons should be tailored to provide variety in instruction: large and small group instruction, problem solving, concept development which includes "hands-on" experiences with manipulatives, drill and practice, and classroom discussion. Expectations should be realistic. Since skill retention may be a problem, frequent monitoring should occur in the form of homework checks and small quizzes instead of only a few major tests.

Calculators should be an integral part of a general math/pre-algebra course. They can be used to motivate students and facilitate the solving of problems which apply basic concepts and to investigate numerical patterns and properties. The course content needs to emphasize concept formation through the use of manipulatives and representational models. It is not enough to concentrate on the computational algorithms. There must also be ample experience with applications to provide understanding of concepts.

At the beginning of the year, topics should be selected which are motivating or novel, and guarantee success for most students. Suggestions for an introductory unit include: whole number patterns; logic puzzles; and activities which apply such problem-solving techniques as "make a table," "guess and check" and "draw a picture." Concepts and terminology can be reviewed through such applications as how to score bowling and dice or board games.

The following topics should be included in all courses designed for the first year of high school math credit. The reader will note that paper/pencil drill on computation has been de-emphasized in favor of an emphasis on problem solving and concept development.

1. Whole numbers: use of place value (multiplication and division by powers of 10), rounding, estimation, patterns, exponents, application.
2. Common fractions: equivalency, order and applications.
3. Decimal fractions: place value, rounding, estimation, equivalency to common fractions and applications, including money-related activities and metric measurement.
4. Ratio and proportion: equivalences of ratios and solution of proportions. Applications could include rates and scale drawings. Pi and its relationship to a circle can be explored.
5. Percent: equivalency to decimals and common fractions, estimation, working with percent.
6. Measurement: emphasis on concepts of length, area, surface area and volume. The use of physical models (cubes, tiles, etc.) is recommended.
7. Informal geometry: emphasis on vocabulary associated with basic shapes. Square roots can be investigated and modeled as lengths of diagonals of squares.
8. Probability: emphasis on experimental probability (coin tossing, dice, etc.).
9. Statistics and graphs: emphasis on interpreting data and graphs.
10. Algebra: emphasis on use of a variable to generalize a result, practice evaluating expressions and formulas, solving simple linear equations.

Resources to supplement existing textbooks are available. A bibliography is listed on pp. 191-193. It is very important at this level that money be budgeted for these resources and for inservice training. It should be the responsibility of the entire math department of a school to provide excellent general math instruction.

For materials useful in developing such a course, see the list of resources appended to this report.

OPTIONS FOR THE SECOND YEAR

Applications of Mathematical Concepts

The second course need not be taken in the year following completion of the general math or pre-algebra course. In fact, it is advisable for it to be offered to students in their junior year of high school since this allows students to have matured both in thinking ability and awareness of their own career choice and decisions they must make as an adult.

Schools should attempt to offer as one option a course designed for students of lower ability or of lower interest level designated in this paper as "Application of Mathematical Concepts."

The remainder of this section describes a course of this type. It should emphasize problem solving and the understanding of mathematical concepts. Quoting from NCTM's "Agenda for Action":

> The need of the student to deal with the personal, professional, and daily experiences of life requires a curriculum that emphasizes the selection and use of these skills in unexpected, unplanned settings.

Materials and problems include mathematics from the sciences, business and consumer world, sports and other hobbies, the social sciences, trade and industry, home projects and maintenance, and travel, as well as problems and patterns from the world of mathematics included for their intrinsic fascination and beauty. The hand held calculator should be used as a constant tool in all class work and tests. Mental arithmetic, estimation and "checking for the reasonableness of answers" needs to be strongly and constantly emphasized.

Instructors need to be aware of the career choices, interests and major means of livelihood in their community. The skill areas in this course can be much the same as topics often found in a general mathematics course. However, the difference is in the sophistication of approach, depth and complexity of problems to be solved, and the expectations by the teacher of a high degree of understanding.

Skill areas include:

1. Use of the Calculator

   This includes working with order of operation, constant function, square root, estimating and then checking answers, analyzing multiplication, division and decimal fractions.

2. Estimation and Approximating Skills

   This includes understanding concepts such as division, fractions, percent, rounding and the distributive property and operations by powers of ten. The unit includes exercises in making reasonable guesses.

3. Mental Computation

The emphasis here is on knowing "multiplication facts" and how an understanding of the distributive property and fractions, percent and equivalencies leads to mental arithmetic shortcuts.

4. Common and Decimal Fractions

This includes a general review which emphasizes visualization of equivalencies and work with place value followed by an abundance of problems which apply these concepts.

5. Basic Algebra

This includes solving simple linear equations, especially proportions, and evaluating expressions including common formulas. Emphasis is placed on applying these concepts to verbal problems.

6. Percent

This unit includes finding amounts, identifying percent parts of the total, understanding percent as a fraction, and decimal/common fraction equivalences. Applications include estimation of tips, calculating interest, discounts, taxes and commissions.

7. Tables/Charts/Graphs

Application problems include reading and interpreting charts, tables and forms (mileage, postage, catalog, tax, banking, telephone rate schedules, etc.).

8. Measurement/Geometry

This includes converting within the metric or English system and applying perimeter, area and volume determination to such problems as paint and carpet coverage, deck building or fencing.

9. Pythagorean Theorem and Triangle Trigonometry

This includes applying the Pythagorean theorem and the concept of trigonometric ratios to construction-related problems, learning to use the square root key and trig function keys on a calculator, and reading tables.

10. Probability

This includes listing the outcome of an event, expressing probability as a ratio of successes to total outcomes and recognizing probability as a predictor for a large number of trials (weather forecasting, principles behind insurance, sporting odds and games of chance).

11. Statistics

This includes collection and organization of data and determination/comparison of mean, mode and median.

12. Career Awareness

Throughout the course an attempt should be made to increase the students' awareness of the use of mathematics in a variety of careers.

For a list of materials useful in developing such a course, see the resources appended to this report.

OPTIONS FOR THE SECOND YEAR

Options for Students Who Have Completed Algebra I

Considering the wide variation in student population and student need, there are a number of courses which can follow Algebra I other than the traditional Geometry/Algebra II sequence which will satisfy the two-year graduation requirement. The group of students addressed in this section of the report are either (1) not interested in the college prep sequence of Algebra, Geometry, Algebra II, or (2) have had limited success in their first algebra course.

Listed below are several nontraditional courses which the task force feels would adequately satisfy the second year of the high school graduation requirement. This is by no means an exhaustive list, but is meant to give the flavor of the variations possible.

EXAMPLE 1: "Applied Geometry"

This course deals with traditional geometric concepts and theorems with formal proof not required. The emphasis is on nontraditional application of these theorems. A course of this type was developed at Medford Mid-High School by Paul Myers, Dennis Dedrick and Gary Morrison.

EXAMPLE 2: "Survey of Math Topics"

This course is built around problem solving as presented by Harold Jacobs in *Mathematics: A Human Endeavor* and around a development of practical trigonometry. An example of a course of this type to follow geometry is being taught at Glencoe High School in Hillsboro, Oregon.

EXAMPLE 3: "Survey of Algebra/Geometry"

This course integrates a review of Algebra I topics with the introduction of basic geometric concepts. An example of a course of this type to follow Algebra I is being taught at Glencoe High School in Hillsboro, Oregon.

EXAMPLE 4: "Senior Topics"

This course exposes the student to a variety of topics (probability, statistics, linear and exponential growth) that have real world practical value at a level below the college prep sequence but above the current Consumer Math level. The topics are sophisticated, but the approach is nontechnical. An example of a course of this type has been developed by Tony Gerlicz at Catlin Gabel High School in Portland, Oregon.

EXAMPLE 5: "Advanced Topics on Elementary Algebra"

This course is a review and extension of several major concepts of Algebra I with an introduction to mathematical topics outside of the standard college prep sequence using *Mathematics: A Human Endeavor* as a foundation. An example of a course of this type has been developed by Mike Bolduan at Catlin Gabel High School, Portland, Oregon.

EXAMPLE 6: "Probability and Statistics"

This course deals with topics found in a standard probability and statistics course but presents them in a manner accessible to students with a limited background in algebra. Lessons are centered around real-world sources and are not textbook dependent. A course of this type is being taught at Hood River High School, Hood River, Oregon.

EXAMPLE 7: "Problem Solving in Mathematics"

This course provides students with specific techniques and methods for solving mathematical problems. The primary objective of the course is to teach the process of problem solving. A course of this type has been developed by the Ohio Department of Education.

As mentioned above, this list is not exhaustive and there are many other appropriate courses which could be used to complete the two-year graduation requirement, such as courses in the area of vocational or technical mathematics and discussed in the "Oregon Mathematics Concept Paper No. 7: Vocational and Technical Mathematics."

In addition to the curriculum papers resulting from the OMEC Curriculum Project, the National Council of Teachers of Mathematics has two recent publications which focus on high school mathematics curriculum. Both are MUST reading for persons serious about updating high school mathematics curriculum so that various populations of students being impacted by the two-unit graduation requirement are offered viable options. NCTM's 1985 Yearbook *The Secondary School Mathematics Curriculum*[1], discusses new curricular directions and innovative courses. The supplement to the 1985 yearbook, entitled *Alternative Courses for Secondary School Mathematics*[2], offers course descriptions and contact persons for such courses, which are being offered in many states.

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[1] *The Secondary School Mathematics Curriculum, 1985 Yearbook*
Christian Hirsch and Marilyn Zweng, Eds.
National Council of Teachers of Mathematics
1906 Association Drive
Reston, VA 22091

[2] *Alternative Courses for Secondary School Mathematics*
Marilyn Suydam, et al.
National Council of Teachers of Mathematics
1906 Association Drive
Reston, VA 22091

GRADUATION REQUIREMENTS: MATHEMATICS

RESOURCE MATERIALS

1. Making Practice Fun: General Mathematics, A Fundamental Approach
   by Ray Chayo

   Addison-Wesley Publishing Company, 1980
   Menlo Park, CA

2. Projects Booklet: Applying Mathematics, A Fundamental Approach
   by Mervin L. Keedy

   Addison Wesley Publishing Company
   Menlo Park, CA

3. Projects Booklet: General Mathematics, A Fundamental Approach
   by Mervin L. Keedy

   Addison-Wesley Publishing Company
   Menlo Park, CA

4. Mathematics for Today (Level Red, Level Blue)
   by Alexander Tobin, Ed.D., etc.

   Oxford Book Company, Inc.
   Los Angeles, CA

5. When Are We Gonna Have To Use This?
   by Hal Saunders

   Dale Seymour Publications
   PO Box 10888
   Palo Alto, CA 94303

6. Newspaper Math
   by Don Fraser

   Dale Seymour Publications
   PO Box 10888
   Palo Alto, CA 94303

7. Developing Skills to Estimation (Book B)
   by Dale Seymour Publications

   Dale Seymour Publications
   PO Box 10888
   Palo Alto, CA 94303

8. Tables and Graphs (Book B)
   by Elaine C. Murphy

   Dale Seymour Publications
   PO Box 10888
   Palo Alto, CA 94303

9. Beefing Up Basic Skills: Fractions, Decimals, Percent (3 books)
   by Mari O'Rourke

   The Math Group
   Minneapolis Public Schools
   396 East 79th Street
   Minneapolis, MN 55420

10. Math Amusements in Developing Skills
    by Alice Clark, etc.

    Midwest Publications
    PO Box 448
    Pacific Grove, CA 93950

11. Word Problems:
    Series Titles: Introductory Word Problems,
    Whole Numbers - 1
    Fractions - 1,
    Decimals -1
    Percents - 1
    Percents - 2,
    Mixed Concepts A-1
    Mixed Concepts B-1,
    Mixed Concepts C-1

    Midwest Publications
    PO Box 448
    Pacific Grove, CA 93950

12. Mini-Calculators in the Classroom
    by Joseph Caravella

    National Education Association
    Washington, D.C.

13. Games, Tricks, and Puzzles for a Hand Calculator
    by Wallace Jud

    Dymay, 1974
    Menlo Park, CA

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14. Calculator Math Problem Solving
    by Edward C. Beardslee, 1982

    Enrich, Subsidiary of Ohaus Scale Corporation
    San Jose, CA 95131

15. How to Develop Problem Solving Using a Calculator
    by Janet Morris

    National Council of Teachers of Mathematics
    1906 Association Drive
    Reston, VA 22091

16. Mathematics in Science and Society
    by Alan Hoffer, etc.
    University of Oregon Mathematics Resource Project

    Creative Publications
    3977 East Bayshore Road
    PO Box 10328
    Palo Alto, CA 94303

17. Handy Math

    (Focus on: Purchasing, Sports, Travel, Map Skills, Earning Money, Managing Money)

    Creative Publications
    3977 East Bayshore Road
    PO Box 10328
    Palo Alto, CA 94303

18. Calculator Explorations and Problems
    by Don Miller

    Cuisenaire Company of America
    12 Church Street
    New Rochelle, NY 10805

19. Advanced, Intermediate, and Beginning Calculator
    by Gerardus Jervordt, et al.

    Cuisenaire Company of America
    12 Church Street
    New Rochelle, NY 10805

Oregon Mathematics Concept Paper No. 7

VOCATIONAL AND TECHNICAL MATHEMATICS

PREFACE

The Oregon Mathematics Education Council (OMEC) launched a mathematics curriculum project in May 1984. Nearly 100 enthusiastic, dedicated volunteers, a majority of whom are classroom teachers, worked on the task forces of the project. These groups worked to develop a set of concept papers relating to major issues of reform in mathematics education. Their ideas are shared now as a set of *The Oregon Mathematics Concept Papers*. They are available as a complete set or as separate papers. You are encouraged to read those papers of special interest and to discuss the ideas with colleagues. Duplicating the papers for such noncommercial purposes is encouraged.

OMEC is a widely representative advisory group, representing teachers at all school levels, counselors, administrators, school boards, business, labor, industry, and parents. *The Oregon Mathematics Concept Papers* are being disseminated by the Oregon Mathematics Project, working with a number of mathematics organizations, and many individuals. The organizations are:

Oregon Mathematics Education Council
Oregon Council of Teachers of Mathematics
Oregon Math Leaders
Teachers of Teachers of Mathematics
Community College Math Chairers
Honors Institutes

For additional copies of *The Oregon Mathematics Concept Papers*, or information regarding speakers and/or discussion leaders, contact:

Mike Morgan, President
Oregon Mathematics Education Council
Linn-Benton Community College
6500 SW Pacific Boulevard
Albany, OR 97321
(503) 928-2361 x467

Ron Morgali, Project Consultant
Oregon Math Project
Western Oregon State College
Monmouth, OR 97361
(503) 838-1220 x345 or x206

Don Fineran, Specialist
Math Education
Oregon Department of Education
700 Pringle Parkway SE
Salem, OR 97310-0290
(503) 378-3566

Jim Stewart, President
Oregon Council of Teachers
of Mathematics
Oregon Institute of Technology
Oretech Branch Post Office
Klamath Falls, OR 97601
(503) 882-6321 x161

The Oregon Department of Education supports, facilitates and coordinates ongoing efforts among these organizations, including the OMEC Curriculum Project. Don Fineran serves as Department liaison to all of the above organizations.

The OMEC would like to thank Al Elkins of Porter Public Relations, Portland, for his help in editing these papers.

OMEC TASK FORCE
on
VOCATIONAL AND TECHNICAL MATHEMATICS

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OVERVIEW

The original charge of this task force was the preparation of a possible position paper for OMEC on vocational/technical mathematics education. In proceeding, the task force first attempted to define vocational/technical mathematics. Some distinctions between the two seemed obvious, and could be made by looking at the students potentially served by each course. Students in vocational mathematics could generally be assumed to have little or no mathematics background beyond general math or algebra. Such students would be taking the course for use in vocational courses or as part of the two-year graduation requirement. Technical mathematics students, on the other hand, would probably have completed geometry and, perhaps, a second year of algebra, as well. They would be in the course as an alternative to the traditional mathematics program.

After identifying the students each course would be aimed at, proposed outlines for each were prepared. These outlines were then sent to all mathematics and industrial arts departments in Oregon public high schools for comment and review. The returns from this survey were evaluated and used as a basis for modification of the course outlines. Through further committee discussion, modified course outlines were prepared. Some topics were eliminated, others added. The topics themselves were reorganized under seven major headings: measurement, communication, computation, basic algebra skills, probability and statistics, basic geometry skills, and reasoning. These major content or topic strands were carried through in each outline.

With these outlines better defined, and the information that little research or work had been done in the area of vocational/technical math, the task force expanded its work, somewhat, to include a more detailed discussion of the content of vocational and technical mathematics courses and to look at what would be needed for such courses to be implemented successfully.

The task force concluded that the courses which appeared to be potentially useful would require a shift in emphasis and philosophy from common, traditional mathematics curricula. This shift would require new materials to be developed and inservice to be made available, if not required for the courses' teachers. To assist this process, amplification and explanation of the course outlines became a necessity. Further, the courses present some opportunities and problems that would need to be addressed in developing the course materials and using them in school districts. An attempt was made to address this as well.

After completing this much-expanded task, the task force is reinforced in its belief that there is a place for these courses in many mathematics curricula. More accurately, there may be a real need for these courses in the curricula. Traditional courses do not address the need of many students for alternative courses that teach mathematics as a practical skill going beyond simple computation. The proposed courses offer that option in programs that preserve their integrity as mathematics. They offer flexibility for the poor student, as well as the advanced student. Additionally, they offer survival skills at a realistic and relevant level.

I. Vocational/Technical Mathematics: A Necessity, Not a Luxury!

In the traditional mathematics curriculum, the K-12 progression is often seen as a quest for the ultimate goal: i.e., precalculus or calculus. Following this concept, we would be accurate in describing first grade mathematics as precalculus 12, beginning the countdown. Unfortunately, when designing or organizing a mathematics curriculum, this perception is frequently paramount. The result is that, too often, our mathematics programs are focused on the relative few who will reach this level. The majority of students in our programs are ignored, as we plan for the very best and ignore the rest. It is becoming increasingly clear that this approach does not adequately equip many of our students with the mathematics skills they need.

Most obvious is the lack of emphasis on practical applications of the various concepts learned. In many cases, word problems are skipped by teachers, because they are "too difficult" for students. Many of the problems presented in traditional materials are so artificial and unworldly as to be of little interest to students. Truly relevant, interesting problem solving and application is absent. The result, then, is a student who may understand the theory, but can do little with it. In many cases, even the "understanding" is superficial. This can be easily verified by talking to industrial arts, physical science, or physics teachers. They are sure to inform almost anyone who inquires that many of their students cannot measure, interpret graphs, insert values into formulas, calculate areas or volumes, nor do they really seem to understand these or other concepts their transcripts indicate they have mastered.

These problems are an inevitable result of abstract math curricula which aim toward a high goal, but do not give realistic preparation to most students. Anyone familiar with Piaget's or similar models of intellectual development knows that the highest levels of reasoning are achieved by only a portion of humanity. Estimates of how many actually achieve Piaget's formal-operational stage range from 20 to 60 percent. Yet many current math curricula seem to expect everyone to reach this level. These curricula follow a tightly defined path, without acknowledging that many students would be better served by programs oriented toward applications and practical use of a limited number of abstract concepts, rather than a theoretical excursion into the world of mathematics.

Awareness of this problem has periodically surfaced, with emphasis on the students of lowest mathematical ability. This has given rise to the "Math for Life" type of class, which seems to follow a cycle of appearance, initial success, decline, and elimination. All too often, such classes turn out to be another class in computations, a subject in which students enrolled in the class have already shown a lack of facility. That a student cannot work with basic concepts of algebra, geometry, or probability, until pencil and paper computation has been fully mastered, seems to be a general assumption. This is another side of the often rigid adherence to the standard mathematics curriculum. Such an assumption ignores the trend toward increased use of calculators or computers for computation.

In Oregon, over 20 percent of students graduating from high school take only one year of mathematics. When those who drop out are included, on the order of 40 percent of students who enter high school in Oregon will have taken only one year of mathematics. For most of these, their last course will either have been a general math course, focusing on computation, a pre-algebra, or an algebra course. In all of these classes, application and practical usage are generally secondary.

With the two-year graduation requirement coming into effect, these students will be confronted with the need, if not necessarily the desire, to take another year of mathematics. This new requirement presents Oregon schools with a marvelous opportunity to address two problems with a single solution. Many of the students who are currently taking a single year of mathematics are enrolled in industrial arts classes. Others, with better mathematics preparation, are also enrolled in such classes. Both groups have similar problems in the industrial arts classes. However much math they know, they seem to be unable to use it in the practical situations confronted in industrial arts classes. Many industrial arts departments have, in the past, offered vocational mathematics classes to address this problem. Often, these have been unsatisfactory, particularly when viewed from the perspective of the mathematics department.

When a close look is taken at the math skills required by the vocational classes, it becomes clear that while a few, such as blueprint reading, are specialized, most are skills that would be useful to people in everyday life. In our standard math curricula, we do not address them explicitly, assuming that our students will be able to "pick them up" if they complete our programs. Some do, but many do not.

The two-year requirement presents an opportunity to offer a course which will address both the needs of the vocational student and the student who formerly took only a single year of mathematics. Such a course could be designated a vocational mathematics course, but would, in reality, be an applied mathematics course, drawing from a variety of areas of the mathematics curriculum, rather than the more traditional courses focused on paper and pencil computation, abstract algebra, or theoretical geometry. Such a unified course could concentrate on practical skills useful to everyone, but of particular importance to the vocational areas.

Vocational mathematics courses can provide a viable second course to those who took only a single math course in the past. They would not focus on computations, nor would they explore the formal or theoretical aspects of algebra or geometry. They would focus on the basic concepts of algebra, geometry, and other fields of mathematics as they apply to the nontechnical everyday world. Computation would play a role, but the use of calculators and other computational tools would eliminate the stress and drudgery often associated with it. These classes could borrow heavily from the vocational education departments and science departments for demonstrations and problems. The emphasis should be on how the mathematics could be used, rather than on the mathematics itself. Accordingly, we have prepared an outline of a possible vocational mathematics course, which is discussed and expanded upon later in this paper. This type of class could, even in the case of small schools, provide the second year program for students taking the minimum mathematics requirement. In larger schools, it could be an alternative second course, as well as a course designed for students taking a program with concentration in the vocational areas.

The lack of application and practical problem-solving in most current mathematics curricula impacts not only those students taking minimal math or in the vocational areas, but more advanced students as well. Often those students who are not ready for calculus, precalculus, analytic geometry, or other classes beyond Algebra II have nowhere to go in the available curricula. Additionally, those interested in technical fields or careers, particularly those not planning on a four year college program, have few options in many current mathematics programs. A reasonable course for these students could involve many of the concepts in the proposed vocational mathematics course, as well as more advanced topics. Concepts included in such a course, which were also in the vocational course, might be amplified upon, or covered with a greater mathematical sophistication, consistent with the background and abilities of the students. The other topics, such as manipulation of algebraic equations, including quadratics, would require substantial mathematical sophistication for a high school student. This type of course might accurately be identified as technical mathematics.

The technical mathematics course should emphasize use of moderately high levels of mathematics in realistic, technical applications. One example would be using techniques for simultaneous solution of equations to analyze an electric circuit. Another would be the use of log-log paper to analyze experimental data and derive an equation. The focus of the course should be mastery of the skills through use in realistic applications, not sterile textbook exercises written 15 years ago. Discussion of such a course is also included in this paper.

The two proposed courses, vocational mathematics and technical mathematics, have definite similarities in content and approach. Before discussing them in detail, it seems reasonable to define them with some degree of precision.

Vocational Mathematics

This course is intended to meet the second year of the two-year graduation requirement, and/or equip students with the basic mathematics skills needed in vocational education courses. The underlying assumption is that students taking this course will have fairly poor mathematics backgrounds, having progressed no further than algebra, with many having taken no mathematics beyond general math/computation. With this assumption, it will be necessary to introduce concepts in algebra, geometry, and other areas with little theoretical development. Many concepts will be introduced as "techniques" or "skills" with emphasis on how and where they can be used, rather than their origin or further implications. Application, demonstration, and use will be the focus. Linkage between abstract concepts will not be emphasized. This course might be reasonably considered a survival course.

Technical Mathematics

This course is intended as an elective, an alternative to analysis, senior math, precalculus, calculus, or similar courses. It is intended for students who have completed courses in algebra, as well as geometry. Such students may reasonably be assumed to have a fair degree of mathematical sophistication. They should be able to solve and manipulate equations, including second order equations. Techniques of simultaneous solution and graphing should have been learned, though not necessarily remembered in detail. Basic geometric concepts, including congruency and similarity, should be well understood. The basic trigonometric functions should be part of the students' working knowledge. With this basis, the course should emphasize practical application of these concepts in technical areas, using examples and experiments from vocational, technical, and scientific fields. While new material and skills should be developed, and other skills presumed to have been developed but never explicitly taught should be discussed, much of this course should involve a review of topics with a view to application. Use of calculators and computers should be integrated in the topics.

As may be obvious from these two course definitions/descriptions, we perceive the vocational math course as providing much new material to students, while the technical math course will provide less new material, but will focus on helping the student review and use previously obtained knowledge and skills in a practical context. It is not our intent to completely write these two courses. There are many ways in which the topics we believe they should contain can be presented. Our purpose is to provide an outline of the topics to be covered, some explanation of the rationale for the choice of each, and some suggestions for approaches and materials. Local orientations toward certain definite vocational or technical fields may provide opportunities for greater specificity than would be appropriate in our more general presentation.

Choice of topics and the related suggestions originated with this task force of OMEC, and have been reviewed and discussed with educators in both mathematics and vocational education, at both the secondary and community college level.

II. Topic Outlines--Survey Results

After the original topic outlines for the vocational and technical mathematics courses were prepared, the task force felt that it was necessary to obtain input from teachers in the fields of mathematics and the industrial arts. We developed what we thought were reasonable courses. Any further work, however, would be pointless, unless our ideas were regarded as reasonable by those who might find themselves teaching such courses. Since part of our goal was to design courses which would serve the needs of the industrial arts departments, as well as the mathematics departments, evaluation of the proposed outlines by both groups was seen as necessary.

The consensus of the task force was that as broad a group as possible should be surveyed. After consideration, it was decided that a survey form should be developed and sent to the mathematics and industrial arts departments of all Oregon public high schools. This survey form was based on the topic outlines. Recipients of the survey were asked to indicate how vital each of the topics listed in the outlines would be in each of the two proposed courses. The survey instrument, detailed results, and a key to the responses are all included in a draft version of this paper, which is available from Don Fineran.

Response to the survey was, of course, voluntary. It would be reasonable to assume that those who perceived the greatest need for these courses would be most likely to respond. With approximately 240 schools receiving the surveys, we expected a 10 to 15 percent response, based on prior experiences. The actual returns exceeded this by a gratifying amount. Over 60 mathematics departments, or more than 25 percent responded. More than 50 industrial arts departments returned their surveys. The task force is grateful for the time and effort expended by the respondents.

In analyzing the results, a few basic criteria were followed. Response 1, essential; 2, very important; and 3, important, can all be assumed to indicate that a topic should be among the topics included in the course. Response 4, not vital, indicates a topic that could be included as enrichment, but is not part of the "core" of the course. Response 5, not useful, indicates that topic should not be included in the course. If responses 1, 2, and 3 are given on 75 percent of the surveys or more, it seems reasonable to assume that a topic had broad support and should be included.

Using this criteria to examine the results of the survey on the vocational mathematics course, we saw that most of the topics suggested are perceived as appropriate by both mathematics and industrial arts teachers. Two exceptions are use of verniers and specialized measurement units. Both of these exceed the 75 percent approval level in industrial arts responses, but fail by a small margin to reach that level on the mathematics department returns. Similarly, the reading of spread sheets was approved by 81 percent of industrial arts teachers, while only 73 percent of the mathematics teachers indicated approval.

The difference in the perceptions of those two groups on the three topics may possibly be due to the fact that the course topics have been deliberately oriented toward application. Since this is not the prevailing orientation in the mathematics community, it may explain the slightly lower approval rating.

Even though these three topics fail to meet the 75 percent approval criteria, they come very close to that level, indicating substantial support for their inclusion. Two other groups of topics fail to meet the criteria by larger margins. Exponentiation is one of these. While meeting the approval criteria on the industrial arts responses, basic concepts and use in formula received approval percentages of 67 percent and 72 percent, respectively, from mathematics teachers. The exact reason for this disparity is not certain.

Somewhat surprisingly, probability and statistics received poor support from both groups. Averages received approval from 79 percent of the mathematics responses and 66 percent of the industrial arts surveys. Three other topics, chance events, sampling, and data reliability, all received less than 50 percent approval from both groups. This indicates support for these topics is weak, and will be discussed further in the section of this paper on issues.

Comparing the responses of the industrial arts departments with those of the mathematics departments, a pattern emerges. On most topics, the industrial arts responses indicate a higher approval than the mathematics responses. In particular, the percentage of responses indicating that topics are "essential" is consistently higher. This indicates that we have probably been successful in making the topic outline emphasize practicality.

Responses on the technical mathematics surveys followed a pattern similar to those on the vocational mathematics survey. Very few topics failed to achieve a 75 percent rating on either survey. Items not appearing on the vocational mathematics survey which failed to achieve this level of approval on the technical mathematics survey were logarithms, use of matrices, transformational applications, nontraditional geometries, and extended projects. All of these received approval ratings of less than 65 percent from the mathematics teachers. Surprisingly, these topics received approval levels higher than 80 percent from industrial arts teachers.

As was the case with the other survey, support for probability and statistics was weaker than most others. In general, however, the individual topics did receive approvals of at least 75 percent. Only permutations and combinations failed to achieve this level of approval.

The pattern of higher approval ratings given by industrial arts teachers on many topics is repeated in this survey. If anything, it is more consistent and marked by greater differences, particularly in the percentages of responses indicating that topics are essential.

Both surveys, then, indicate that the topic outlines developed by the task force are generally seen as valid by the mathematics and industrial arts communities. Some of the topics received lower levels of support. We have, however, decided to retain them. A discussion of our rationale for doing so is included in the section of this paper entitled "Issues." Further task force discussion and consideration of comments included on the surveys led to a modification of both the structure and specific items included in the topic outlines.

In addition to the information about the course outline, the survey was also designed to extract information about the number of years of mathematics taken by graduating seniors and the last mathematics course taken by graduating seniors. Our assumption, in designing both courses, was that there were a significant number of students who graduated with only one or two years of mathematics. These students would provide a natural group for our vocational math course. Another assumption was that a large number of graduating students completed no more advanced mathematics course than Algebra I or its equivalent. This would be necessary for vocational mathematics to be a viable course, particularly in the smaller high schools.

Of the surveys returned, not all were identifiable as originating from a single school. Additionally, not all those schools which identified themselves on their survey form completed the sections on "years of mathematics taken" or "last course taken." Those who filled out either of those sections, however, have a total student enrollment of 26,503, a substantial fraction of the total high school population of the state. It is reasonable to assume that patterns observed in this group will be generally followed throughout the state.

The total number of students enrolled in those schools for which data was available on the number of years of mathematics taken was 25,803. In these schools, 11.6 percent of the graduating class of 1984 took only one year of mathematics and 38.2 percent took two years of mathematics. Clearly, our assumption that a significant number of students completed only one or two years of mathematics was valid.

Schools with total enrollment of 25,804 students completed the section of the survey which allowed us to determine the last math class completed by graduating seniors in the class of 1984. Of these graduates, 18.0 percent took a mathematics class below the level of Algebra I or its equivalent as their last class. This includes pre-algebra or the first half of two-year algebra sequences. Completion of both halves of a two-year algebra sequence was counted as equivalent to Algebra I. There were 35.8 percent of graduates who completed math courses no higher than Algebra I or its equivalent. Over half, 51.2 percent, went no further than Geometry. This information validates our second assumption, namely, that there is a large body of students for whom the vocational mathematics course is appropriate, based on their previous mathematics experience.

III. Vocational and Technical Mathematics Course Topic Outlines

The task force revised the original topic outlines emphasizing grouping of topics under common categories. These outlines, it must be emphasized, are intended to be broad and nonspecific. They are merely intended to give an indication of what topics would make appropriate courses. All topics need not be included for a course to be useful. In the case of the technical mathematics course, inclusion of all topics would be impractical in a one-year course. The order of topics within a group are an indication of the priority as determined by the task force and/or the survey results.

SUGGESTED TOPICS FOR INCLUSION IN VOCATIONAL MATH

A. Measurement

1. Measurement Tools
    a. Common, nonprofessional tools
    b. Linear scales
    c. Verniers
    d. Gauges
    e. Nonlinear scales
    f. Volume measurement tools
    g. Tilings

2. Approximation
    a. Measurement
    b. Calculation

3. Measurement types
    a. Linear measurement
    b. Area measurement
    c. Volume measurement
    d. Angular measurement
    e. Measurement
    f. Calculation

4. Units of Measurement
    a. Metric
    b. Specialized in vocational areas
    c. Measurement forms--decimals
    d. Conversions in the same system
    e. English

5. Advanced Measurement Tools--Computers
    a. Basic operation
    b. Computer simulation
    c. Use and selection of software

B. Communication

  1. Reading and Interpretation
     a. Charts
     b. Graphs
     c. Data
     d. Spread sheets (simple or limited)

  2. Oral Communication
     a. Use of correct terminology
     b. Ability to follow instructions (oral and written)

C. Basic Computational Skills

  1. Computation (Four Basic Functions)
     a. Estimation
     b. Calculator
     c. Mental
     d. Pencil and paper
     e. Development of a "feel" for reasonableness
     f. Computer
     g. Significant Figures
     h. Choosing an appropriate method

  2. Fractions, Percents, and Decimals
     a. Use throughout other topics
     b. Identification of when each is appropriate
     c. Use in formulas
     d. Skill reinforcement

  3. Money Problems
     a. General

D. Basic Algebra Skills

  1. Formulas
     a. Identification of appropriate formulas
     b. Use of formulas by insertion of values
     c. Development of knowledge which will aide in locating formulas

  2. Exponentiation
     a. Basic concepts
     b. Use in formulas

E. Probability and Statistics

  1. Basic Probability and Statistics
     a. Averages
     b. Chance events
     c. Sampling
     d. Data reliability
     e. Mean, median, mode

F. Basic Geometry Skills

1. Informal Geometry
    a. Area of regular shapes
    b. Perimeter of regular shapes
    c. Volume of regular shapes
    d. Identification of regular shapes
    e. Vocational terminology/mathematics terminology
    f. Basic properties
    g. Transformations--effect on apparent shape
    h. Visualization
        (1) Similarity
        (2) Congruency
        (3) Construction of models
        (4) Blueprinting

G. Reasoning

1. Problem Solving
    a. Techniques
        (1) Guess-Check-Refine
        (2) Models
        (3) Simplifying
        (4) Patterns
        (5) Making tables
        (6) Working backwards
    b. Materials

2. Critical Thinking
    a. Choosing useful processes
    b. Choosing useful information
    c. Identifying sufficient information
    d. Interpretation of results

SUGGESTED TOPICS FOR INCLUSION IN TECHNICAL MATH

A. Measurement

1. Advanced Measurement Tools--Computers
    a. Basic operation
    b. Computer simulation
    c. Use and selection of software
    d. Spread sheets
    e. Data management

2. Measurement Tools
    a. Common, nonprofessional tools
    b. Linear scales
    c. Verniers
    d. Gauges
    e. Nonlinear scales
    f. Volume measurement tools
    g. Tilings

3. Approximation
    a. Measurement
    b. Calculation

4. Measurement Types
    a. Linear measurement
    b. Area measurement
    c. Volume measurement
    d. Angular measurement
    e. Measurement
    f. Calculation

5. Units of Measurement
    a. Metric
    b. Specialized in vocational areas
    c. Measurement forms--decimals
    d. Conversions in the same system
    e. English

B. Communication

1. Technical Writing
    a. Proposals
    b. Funding
    c. Research
    d. Data presentation/collection format
    e. Collation
    f. Cumulative review

2. Extended project--an extended problem-solving project and formal report

3. Reading and Interpretation
    a. Charts
    b. Graphs
    c. Data
    d. Spread sheets (simple or limited)

4. Oral Communication
    a. Use of correct terminology
    b. Ability to follow instructions (oral and written)

C. Basic Communication Skills

1. Computation (Four Basic Functions)
    a. Estimation
    b. Calculator
    c. Mental
    d. Pencil and paper
    e. Development of a "feel" for reasonableness
    f. Computer
    g. Significant Figures
    h. Choosing an appropriate method

2. Fractions, Percents, and Decimals
    a. Use throughout other topics
    b. Identification of when each is appropriate
    c. Use in formulas
    d. Skill reinforcement

3. Money Problems
    a. Hourly wage
    b. Piece wage
    c. Commission
    d. Taxes
    e. Paycheck deductions
    f. Purchasing and ordering

4. Extended Calculation Skills
    a. Use of scientific notation
    b. Use of trigonometric ratios to solve geometric problems
    c. Logarithms

D. Basic Algebra Skills

1. Formulas
    a. Identification of appropriate formulas
    b. Use of formulas by insertion of values
    c. Development of knowledge which will aid in locating formulas

2. Algebra Skills
    a. General review
    b. Manipulation
    c. Exponentiation
        (1) Basic concepts
        (2) Use in formulas
    d. Simultaneous solutions
    e. Use of matrices

E. Probability and Statistics

1. Basic Concepts
    a. Averages
    b. Chance events
    c. Sampling
    d. Data reliability
    e. Mean, median, mode

2. Basic Statistics
    a. Inferences
    b. Quality control
    c. Data
        (1) Patterns
        (2) Interpolation
        (3) Extrapolation

        - d. Curves
            - (1) Patterns
            - (2) Interpolation
            - (3) Extrapolation
        - e. Diagramming and counting procedures

    - 3. Basic Probability
        - a. Permutations
        - b. Combinations

F. Basic Geometry Skills

    - 1. Informal Geometry
        - a. Area of regular shapes
        - b. Perimeter of regular shapes
        - c. Volume of regular shapes
        - d. Identification of regular shapes
        - e. Vocational terminology/mathematical terminology
        - f. Basic properties
        - g. Transformations--effect on apparent shape
        - h. Visualization
            - (1) Similarity
            - (2) Congruency
            - (3) Construction of models
            - (4) Blueprinting

    - 2. Extended Geometry Skills
        - a. Similarity
        - b. Congruency
        - c. Indirect measurement
        - d. Pythagorean Theorem
        - e. Trigonometric ratios

G. Reasoning

    - 1. Problem Solving
        - a. Techniques
            - (1) Guess-Check-Refine
            - (2) Models
            - (3) Simplifying
            - (4) Patterns
            - (5) Making tables
            - (6) Working backwards
        - b. Materials

    - 2. Critical Thinking
        - a. Choosing useful processes
        - b. Choosing useful information
        - c. Identifying sufficient information
        - d. Interpretation of results

3. Analysis
   a. Data
      (1) Patterns and rules
      (2) Interpolation
      (3) Extrapolation
   b. Curves
      (1) Analyze graph to determine relation between variables
      (2) Interpolation
      (3) Extrapolation

IV. Topic Outlines--Amplification and Discussion

The first major topic of the proposed vocational and technical mathematics courses deals with *Measurement*. This is one of those topics that we all assume students who reach high school have mastered, at least at a minimum competency level, but practical experience of classroom teachers indicates otherwise. While most students may know the basic English units, their mastery of the metric units is minimal. Even within the units they have "learned," vocational and science teachers often discover, to their dismay, that this knowledge involves ability to measure whole units, such as inches, only. Fractions of units are a mystery. The actual value of the individual markings on various measuring devices are a mystery to a majority of secondary students. This makes it impossible for the students to really measure.

Too often, in elementary mathematics, we skim over measurement quickly, giving us more time to do "real" mathematics. The result is an assumption on our part that our students have learned how to measure, when in reality they have not. This is unfortunate, since one of the most basic skills used by everyone is that of measurement. It is impossible to conceive of working in the vocational or technical areas without the use of systems and tools of measurement. Further, in the life experiences of virtually everyone, measurement is an ever-present activity, used for everything from cooking to buying rugs, clothes, or a house. It is the most generally applicable of "vocational/technical" skills.

Because of this, it is the general topic through which so many of the other topics may be developed. Computation, reading and interpretation, use of formulas, fractions and decimals, informal geometry, problem solving, and other topics can all be developed through the use of measurement skills. For that reason, in teaching a vocational or technical mathematics course, little time may be explicitly devoted to this topic. It is essential, however, to insure that it is developed. Equally essential is the requirement that the development be integrated, rather than discrete. Such an approach will develop the greatest understanding with the least waste of time.

In developing units to teach measurement for a vocational or technical mathematics course, it is essential that we follow the lead of our colleagues in the physical science courses. Practical experience using the units and the tools is the only reasonable way to insure that students actually understand them. As is true with language, actual use is the only true proof of mastery and the surest way to develop it. We must not, in such a course, worry about disciplinary compartmentalization. What is effective must be our only concern.

Measurement Tools

Most students are familiar with measurement tools for linear measurement that are marked with linear scales, such as the foot ruler or yardstick. These courses should expose them, in a practical way, to as many measuring tools as is practical. General rules for using scales should be developed. They should be applied in measuring the dimensions of actual objects. The results and accuracies determined this way can be compared to more sophisticated devices, such as the vernier caliper and micrometer. Particular attention should be paid to the use of vernier devices, which have broad application in many fields. Comparative accuracy in measuring small objects with a simple ruler, a caliper, and a micrometer can provide excellent practical experience.

All of these devices, however, are essentially linear scales. It is also desirable that students learn to interpret readings displayed on nonlinear scales. Many such scales are in common use in the trades and elsewhere. Electrical multimeters provide a particularly useful and simple way of teaching use of nonlinear measurement scales. When examining the use of formulas, actual applications and use of Ohm's Law can both test the student's ability to apply formulas and to use nonlinear scales. Most multimeters have one or more functions displayed nonlinearly.

Such use will also develop students' ability to read gauges. Many are linear, but some are not. In general, displays or demonstrations should be set up which will allow students to practice and demonstrate their ability to interpret gauge readings, following the basic rules for determining the calibration of measuring devices developed earlier.

Approximation

The role of approximation in measurement and calculation is poorly understood by most students. That any measurement is itself an approximation is not at all apparent to the population at large. Students should, through use of successively more accurate measuring devices, become aware of this. The role of this in assembly of parts and in the ordering of raw materials can be examined through problems in which tolerances are discussed. A practical activity, such as the number of six-inch blocks which can be obtained from an eight foot two-by-four is an excellent way of illustrating this concept. "Guesstimates" of sizes of objects followed by actual measurement develops both skills.

Approximation in calculation is also a desirable skill. This is an essential skill for determining "reasonableness" of answers. It can be developed by rapid oral drill followed by actual calculation, either hand or calculator, or by estimation followed by experimental verification, as in the case of volume calculations. In either event, this skill should be reviewed frequently, particularly when extensive calculations are being carried out. This is an excellent motivator for review or mastery of the multiplication facts.

Measurement Types

In looking at this topic, the varieties of measurement, it is essential to recognize that most students really only have experience with and are reasonably competent with linear measurement. Experience with area, volume, and angular measurement is minimal. An excellent illustration of this is the question included in the popular board game, Trivial Pursuit, which asks players to differentiate between two square miles and two miles square. Many "well educated" players have difficulty explaining the difference. Experience both in measuring and calculating areas and volumes must be developed. This is particularly easy to do if integrated with the use of a variety of measuring tools and developing a sense of accuracy. A number of activities relating to this can be found in physical science textbooks. Such activities are excellent ways of developing these skills. These activities usually focus on small scale measurements. Larger scale measurements can be carried out by having students measure rooms at school or at home, drawing floor plans, etc.

Most of the students enrolled in vocational mathematics will not have taken Geometry. Accordingly, it will be necessary to teach them the definitions and techniques of angular measurement. A review of this would be in order for technical mathematics students as well. Again, this should be approached by measuring real physical structures, rather than diagrams in a book or on a work sheet. If integrated with the work in informal geometry, the depth of knowledge of both topics can be increased.

Units of Measurement

Most students will have some proficiency in the English system of units. In developing the use of measurement tools and the various types of measurement, other units should be developed. Chief of these, with the gradual metrication of America, should be the metric system. Other quantities, such as torque, which is measured when tightening a bolt on an engine, or board feet, used in the construction and timber industries, should also be developed. Students should be made aware that systems of units are arbitrary, devised for convenience in measurement. By physical comparison they should be encouraged to develop a "feeling" for various systems. Again, this can be developed by comparative measurement of objects by a variety of devices.

Advanced Measurement Tools--Computers

This topic is given highest priority among the measurement topics in the technical mathematics outline. While important, it is of lower priority in the vocational mathematics course. Computers as tools, rather than for programming, should be the focus of their use in these courses. For the type of student who would be taking the courses, computers will be a constant factor in life and work. Use as word processor, test instrument, or simulator will become increasingly common. Students who will have to function in that work/school environment can be prepared for it in this class. They can be trained to be "computer literate" in a truer sense, rather than the narrow sense of the past. This means they should be trained in use and selection of software, including data bases and spread sheets, as well as basic operation. This training can not be exhaustive, but can be integrated with some of the other topics, including the other measurement topics. Emphasis must be on use of the many capabilities of the computer.

In addition to computers, other advanced tools should also be used to make the students more generally competent in measuring skills. One obvious device which suggests itself is the oscilloscope. Learning to use it to measure frequency and voltage will give students some exposure to the use of "high-tech" devices, while at the same time further developing their understanding of measurement.

Communication is the second major topic in the course outlines. As the surveys indicated, however, it would be one of the highest priorities to industrial arts teachers. Communication is a skill that needs little justification or explanation for a language arts teacher, but for a mathematics instructor, an explanation is definitely needed to justify it as part of a mathematics course. It is not an issue to mathematics teachers that mathematics has a language of its own. What causes a problem is that the language of mathematics is not always the language of the vocations. For example, the mathematical concept of slope is an integral part of most mathematics programs. But words like "grade" and "pitch" are more commonly used for the same concept in certain vocations. The vocational/technical math courses should include the language of the vocations and the language of mathematics. To exclude one language would be contrary to the intent of the courses: making mathematics an integral part and a functional tool of our students.

In looking at communication, it is important that more than the use of common language be considered. In all the technical and vocational areas a broad variety of communication skills is needed. Communication should not be confined to writing skills. Reading, speaking, and listening are just as important. Both courses need to develop skills in listening: taking and following directions, taking orders over the phone, listening to speakers, or listening to taped materials. Reading assignments should go beyond the textbook. Exercises, reference materials, magazines, maps, charts, tables, and work orders are just a few of the materials which might be used in the vocational/technical world, making them appropriate for the courses. Making oral reports, answering customers' questions, working in small-group planning sessions, interviewing for jobs, or speaking on the phone are additional communication skills that will be needed and can be worked on in the courses.

Technical Writing

All of the many aspects of technical writing may include mathematical skills directly or indirectly. Actual mathematical computations or presentations may be part of the writing. Alternatively, the writing may be simply to present reports.

Proposal is another word for "bid" to contractors, as is "budget" to the homemaker. Whatever the vocation, a proposal of some name is used in a variety of fields. Often these proposals require more mathematical computations than simple estimates, and can become the life blood of a profession's success. Mistakes in calculation, poor interpretation, or unintelligible presentations can result in loss of work and money. Included in proposals, in certain circumstances, may be applications. Some applications may require specialized writing, either financial statements or legal documents. Funding statements may vary from reconciliation of checkbooks to a financial statement for a bank. Preparation of a tax statement is another funding statement.

Research is no more than collecting data. Calculations are often needed, however, for abstract measurements. In presenting the data or calculations, a variety of formats, ranging from charts and graphs to spread sheets, may be considered. Combinations of different formats are necessary for different readers. The collation of data and the order and clarity of the information may be the difference which brings success or failure. Computers can assist in this, but they cannot evaluate, draw conclusions, or report them.

Summative evaluations are not just answers, but may include forecasts and recommendations. Development of a conclusion and the reporting of the process in a concise statement are expectations of technical writing. These skills can and must be developed, particularly in the technical math course.

Extended Projects

Extended projects are a key part of the technical math course. They are multi-day activities. Problems that require more than one level of technical writing and may require the student to interpret and develop a process for solving the problem, execute the strategy, and report the results over a multi-day period are examples of what this task force feels should be the primary medium of all instruction. Extended projects provide the vehicle within which much other content may be developed.

Projects may involve the whole class in building a house or setting up a mock business. Students going out and collecting the information in a "market basket survey" and reporting the results in the school newspaper is another example of an extended project. Other projects may involve work in the community. The extended project can bring mathematics into the community and make mathematics a tangible tool. Mathematics through extended projects gives the students a feeling of ownership and enhances interest. It also gives students an opportunity to develop reasoning skills.

Reading and Interpretation

In the information age, more and more information is being processed. This information appears in at least five forms. Written form is no more than reading prose. Charts may include blueprints, maps, survey plots, schematics, or diagrams. Graphs are grids that compare one variable of data against some other. Data, in the context of this paper, are tables of information, such as population counts, trigonometric tables, conversion tables, or other examples. The spread sheet is much like a table, except that it also includes summative results.

Students should be given the opportunity to work with each of these ways of representing information. In traditional mathematics courses, only a few are usually featured. An important part of the vocational/technical math courses, however, is the use and interpretation of the information. Included in this must be extraction of additional information by a variety of techniques.

Oral Communication

One of the most critical communication skills, yet the most neglected in the mathematics classroom, is oral communication. Today's classroom is not much different than the television in the home. Students sit in front of their teacher listening to the chatter and watching the electrons flow by. Those same students do not practice the skill of oral communications with their parents or their teacher. Most teachers are appalled by the unintelligible or nonfactual conversations they have with their students. For the most part, mathematics education has not addressed this problem.

Students need to practice their oral communication. They need to be able to speak correctly and report information accurately. Answering the phone is a good exercise. Taking orders over the phone, in which they get nine or ten pieces of information and are then asked to report them can be amusing, but it develops a number of communication skills. Written work is not sufficient. Much of what the vocational/technical math student does in later life will require good oral communication skills.

The third of the major sections deals with *Basic Computational Skills*. The inclusion of this section is necessary for the curriculum to be complete. It is not listed first in the outline, because it is not the "heart" of the course. Our intent is that the computation skills should be secondary to the content of the courses. That a student can work with the basic concepts of algebra, geometry, measurement, and statistics without the mastering of paper and pencil computations is an increasingly accepted idea. Alternative techniques of computation, such as with a calculator, will be more generally used. We would suggest that the instruction in the four basic operations be limited to short presentations when the skills are seen as most appropriate for solving the practical problems.

Computation

The listing of estimation, calculator, and mental computations before pencil and paper computations indicates our stress on the use of mathematics and not on the mathematics itself. To do arithmetic today, mental operations are best for obtaining quick approximations, calculators are tools for computations, and computers are tools for repetitive calculations. Few real life application problems are done with paper and pencil computations. More emphasis is given to the reasonableness of an answer in many cases than the exactness of the answer. Without the time consuming paper and pencil exercises, more instructional time can be given to choosing an appropriate method or to covering other content.

Fractions, Percents, and Decimals

This section is included because of the use of these skills in virtually all other topics. It is necessary for the students to understand the concepts, so they can choose the appropriate form needed for an answer. Use of these skills in formulas will be very common.

Money Problems

Both proposed curricula include the study of money problems. Several types of problems are listed in the technical math outline that may also be covered in the vocational math course. To be able to compute hourly wages, piece wages, and commission is an important skill for any worker. It is also necessary for the wage earner to be able to figure taxes and paycheck deductions. The skills of ordering or purchasing are required in most technical occupations and are probably most appropriate in the technical course.

Since the focus of the courses is the mastery of skills through use in realistic applications, the technical math course includes some extended calculation skills. The understanding of scientific notation and logarithms gives the student the ability to solve more advanced and realistic application problems. The trigonometric ratios provide the means to solve a wide variety of problems, from phase angles in complex AC circuits to stress problems in mechanical engineering.

In both classes, it should be fully understood that mathematics is being approached as a tool for solving problems and not an end in itself. It is important that the arithmetic skills be seen as a means to an answer and not the answer. This must be emphasized for both the student and instructor. It is essential that these courses not become endless exercises in pencil and paper computation.

Section four of each course outline covers *Basic Algebra Skills*. It is not the intent of these curricula to explore the formal or theoretical aspects of algebra. It is the intent that certain algebra skills be taught, because of their usefulness in solving practical problems. The algebra skills that are included should be approached as informally as possible, and the emphasis should be on their use and not on their mathematical significance.

The ability to understand and use formulas is a very important practical application of mathematics. Considerable time should be spent on the identification of the appropriate formula. Students should be able to select the formula that is necessary to solve problems involving the perimeter, area, and volume of geometric figures. They should also be able to use formulas that compute velocity or resistance. Students need to understand the concepts of formulas so that they can use any formula that is given to them. The inclusion of exponentiation in the curriculum outline is based on the perception that exponent concepts are necessary in the solving of many formulas.

The technical math outline includes more algebraic topics. Specifically, it includes general algebra skills, as well as more complex topics such as techniques for simultaneous solution of equations and the use of matrices. These skills are particularly useful in electronics-related applications. Since it has been assumed that the students in technical math would have more extensive mathematics, these more complex topics are appropriate. Many of these topics have already been covered by those students in other mathematics classes. They constitute additional "tools" the students have at their command.

Section five of each course outline covers *Probability and Statistics*. Though rather unpopular in the survey, we feel this topic deserves considerable attention. In high school math classes, we generally assume that students have at least a minimal competency in this area, when in fact many of our students do not know how to compute an average or the simple probability of an event. Often, probability and statistics is a topic found in the last few chapters of mathematics texts and is omitted by teachers when picking and choosing topics to cover as the school year is winding down.

We feel that probability and statistics is not only a basic life skill, but is one which is used frequently in an on-the-job setting. A student should know the difference between, and how to compute such values as mean, median, mode, and probabilities of chance events. Additional skills should include basic ideas of sampling. Most importantly, students should have experience at interpreting statistics. This would, of course, lend itself to much more detailed treatment in the technical math course.

The sixth major topic of the course outlines deals with *Basic Geometry Skills*. This section, like the basic algebra skills section, is not intended to explore the formal or theoretical aspects of geometry. The emphasis should be placed on the practical aspects of the application of geometry skills.

The informal geometry skills are tied closely to the basic algebra skills. In addition to teaching the concepts of perimeter, area, and volume, when and how to use the appropriate formulas should be taught. Because of the applied nature of these courses, there is a need to teach specific vocational terminology, as well as the visualization and transformation of solids.

In addition to the informal skills, the extended skill concepts of congruency and similarity should be taught. These skills are essential when using scales in constructing models and blueprinting. The technical math class, serving a higher caliber of mathematics student, should also cover the right triangle concepts of trigonometry and the Pythagorean Theorem. Again, the emphasis should be on practicality and application, rather than theory.

The last major section is *Reasoning*. Reasoning is a skill that is not unique to mathematics, but part of the foundation of every discipline. Reasoning is not a topic that is best developed by itself, but should be developed in unison with other mathematical topics. This, then, is the problem. At times, other topics or skills are emphasized as being mathematical, while reasoning skills are not taught. When time becomes a factor, the reasoning is often excluded in favor of the more traditional mathematics skills. It is imperative that whatever mathematical skills are taught, reasoning skills be involved in that process. Developing the skill of good reasoning takes time.

Reasoning can best be identified as having four essential sequential skills: (a) formulating the problem, (b) analyzing the problem, (c) finding the solution, and (d) interpreting the results. These four skills make up the structure of what is often referred to as problem solving and critical thinking in mathematics.

Problem Solving

What one considers the problem solving aspect of reasoning is the analysis of the problem and finding the solution. So that there is no misunderstanding, problem solving is not just "word problems." In analyzing the problem, the student needs to identify features central to the problem and plan a strategy to deal with the different features. Here, the "Problem Solving Materials" and "TOPS" materials have been successful. These materials address the different strategies: guess-check-refine, creating models, simplifying the problem, making patterns, making tables, working backward, estimating, drawing diagrams, listing similar elements, and breaking the problem into manageable parts.

Problem solving is a process, not a skill. However, mathematical skills, thinking skills, and often interaction skills are required for the process development. Solving problems requires using one's skills in unexpected, unplanned, and often nonroutine situations. It is the study of how to plan a strategy or activity that can be applied or learned in the solution of a problem. The transfer of problem solving skills from one situation to another, and the application of such skills in decision making processes, are necessary in order that individuals may deal successfully with the world around them.

The "finding solutions" phase is often the primary focus of mathematics classes. Too often, the "number crunching" does not give way to simple logic and overshadows the process of reasoning. This phase or skill, however, should not dominate the first two phases, nor the last phase of reasoning. The finding solutions skill requires mathematical skills which courses often focus on, resulting in that process becoming the sole focus of problem solving. At times, the development of the mathematical skills for finding a solution create students who are not aware that some problems cannot be solved, while others may have more than one solution.

Critical Thinking

The critical thinking part of reasoning is the formulation of the problem and the interpretation of results. The formulation of the problem is difficult for students who have only used the textbook approach. This phase of reasoning needs to become part of the student's development, just as does finding the solution. Students must learn to ask questions, clarify relationships, and determine what information is needed. Once the problem has been solved, then the student needs to interpret the results. Interpreting the solution requires a review of the problem, its constraints and solutions, judging the validity of translations, and checking for accuracy. The interpretation must also involve making correct generalizations and applying the results to solving more sophisticated problems. Finding answers is often more easily tested and can be structured more easily than critical thinking. But time must be provided for students to step back from the problem and evaluate the process.

Critical thinking cannot be treated as content that can be taught as a discrete block of information. As a process, the strategies that must be taught should permeate each and every strand of mathematics education. It is here that most applied mathematics courses fail. The student must be provided enough time to evaluate the best strategy, after trying different methods. Too often the textbook's method or the teacher's strategies are the only processes presented.

Analysis

Analysis is only projected as a topic in technical math. This is due to the greater sophistication of the techniques involved. There are extended methods beyond the simple techniques of problem solving, which are used to help analyze problems. With the development of computers and the study of matrices, large blocks of data can be analyzed. Spread sheets and data management systems are examples of programs for which students must develop rules for different patterns of numbers or shapes. The use of calculators and computers has provided alternative methods for interpolation and extrapolation of data. So, also, the computer can be used to analyze, graph, and determine the relationships that exist between the variables. "Green Globs," "Computer Graphing Experiments," and "Graphical Analysis II" are excellent software materials for enhancing students' understanding.

V. Issues

Arithmetic Skills

It is the position of this task force that arithmetic skills should not be the emphasis of either the vocational or technical math classes proposed. This should be clear from our de-emphasis of pencil and paper computation in favor of mental and calculator computation. We feel that there are many valuable math concepts that can be studied, even if the student has not mastered all the basic computational skills. Indeed, it is necessary to help students see the practical applications of mathematics. (This is particularly critical for those students whose background or success in mathematics is at a low level; in short, many of those for whom the vocational mathematics course is suggested.) This course can assist them in strengthening their math skills through practical use, but only if arithmetic is not emphasized. The study of arithmetic skills should be integrated into the topics covered and not separated from them.

It will be essential to have an administrator or department chair monitor the teaching of the vocational math class to insure that it does not degenerate into just another class in computation. Careful attention should be given to the selection of the teacher for this class. It should not be considered a class for "misassignment."

Probability and Statistics

The survey results showed very weak support for the inclusion of topics in probability and statistics in the two proposed courses. This is in opposition to the position taken by both OCTM and NCTM. It is the position of the task force that the positions taken by the professional organizations are correct. Basic consideration of probability and statistics is not only desirable, it is essential.

In no way is the task force advocating detailed use of probability and statistics in the way most teachers, particularly mathematics teachers, experienced the subjects in college. What we are advocating is teaching the topics which are consistent and appropriate for the mathematics level of the students. These ideas are particularly essential for the vocational mathematics students. Of low mathematical sophistication, they are the precise target of the often misleading and even deliberately false statistics sometimes used in advertising or the sale of products. Some exposure to the topics will provide them with information to make more intelligent choices.

Technical mathematics students, with their greater mathematical sophistication, are likely to engage in activities which will make practical use of probability and statistics. For them, the earliest exposure and use is vital, so that they may begin to integrate it with their other reasoning and mathematical skills.

Two-Year Technical Course

The outline for the technical mathematics course clearly contains far more material than would be reasonable in a single-year course. One option would be to offer a course dealing only with selected topics. This, however, opens the possibility of some key topics being left out entirely. An alternative is to offer the course as a two-year course or sequence. This would allow adequate time to be spent on all topics. Ideally, students could enter in either segment of the course. Rather than offering both years of the course each year, only a single-year course might be offered each year. This would allow students to take both or either halves of the course. Clearly, this option would be a possibility for only the larger schools.

Textbooks

At the current time, there are no textbooks which would be suitable for the two courses the task force has outlined. While there are a variety of texts which claim to be vocational, technical, or vocationally oriented math courses, most tend to be texts in applied problem solving. While a few do address some of the other topics we have outlined, most deal with only a minority of them.

It would be necessary, for the courses to be viable, to develop materials for the courses. Reason dictates that such development be carried out through a publisher or a consortium of teachers and/or districts, rather than being left to individual teachers at the local level. It must precede any attempt to actually implement such a course. Without well-conceived materials, the courses would fail both the students and the course goals.

Inservice

Assuming that materials, including a textbook or syllabus for the two courses outlined, are prepared, it will still be necessary to provide some inservice opportunities either through OCTM, TOTOM, Oregon Math Leaders, or some other body involved in mathematics education. It is important to note that in addition to the proposed changes from traditional content, the attitude and orientation implicit in the course outlines is far different from that of traditional mathematics courses. Unless they can be transmitted to the teachers of the courses, the courses cannot be successful.

It is obvious that any nonmathematics teacher who would be teaching the course would need and benefit from an inservice course. The fact that the philosophy of the proposed courses is far different from more traditional mathematics curriculum makes it reasonable that mathematics teachers assigned to teach the course go through some inservice, as well. All teachers teaching either the vocational or the technical mathematics courses should take an inservice program before beginning to teach it. Materials are not sufficient. The philosophy is perhaps the most important part of the course.

Problems of the Small School

With the increase in specific graduation requirements, all districts, but particularly the small ones, feel the pressure on their course offerings. Each specifically mandated class reduces other options for students. In small schools (less than 300), this leads to a gradual elimination of electives in all areas. The two year mathematics graduation requirement is expected to have some impact of this sort. The fact that no specific course is mandated, however, gives a little more latitude for action.

Teachers and administrators from small schools, when attending conferences, must look wistfully at the many forward looking programs they hear about, knowing that the lack of flexibility their small enrollments cause gives them little chance to try those programs themselves. This need not be the case with the vocational mathematics course. While the technical math course is probably not practical for small schools, the vocational course provides an answer to the problems presented by the two year requirement, while also presenting an opportunity to develop a very creative math course. According to our survey results and the recommended level and approaches of the vocational math course, it would be appropriate for most of those students needing a second year of mathematics. It solves the problem of what to do for them in a way this is useful to them, rather than the more common traditional answer of more computation or algebra at a slower pace. Additionally, it is a viable course in practical use of mathematics for those students who have already met the two year requirement. Since many small schools can barely manage to offer something approximating the normal math curriculum, the vocational math course offers an option for those students who do not wish to move on in a traditional mathematics program. It is valuable to students with strong industrial arts orientation, as well as those who do not fit in traditional curricula. Since it also serves those who need a second year, it is an option which can be very attractive, because of the multiple purposes it serves in a small school. While filling a mandated need, it also provides options for other students.

Staffing the Courses

As more attention is being paid to education, and math and science education in particular, much has been said about the number of mathematics classes taught by nonmathematics teachers. The tendency has been to get quite upset at the thought that people with little or no mathematics background teach math courses. In the case of the two proposed courses, however, many of the mathematics teachers currently employed would not be suitable.

It is the opinion of the task force that a commitment to mathematics is essential for the course. A math degree is not, however, a necessity, nor is preparation specifically as a math teacher. Many of the skills necessary in both courses are not those skills which math teachers have spent a great deal of time developing. They suffer from the same orientation and experience in real-world application. Additionally, the technical math course emphasizes communicating and writing to a degree which is substantially greater than that which most math teachers would be comfortable or competent with.

This suggests that perhaps we do not envision these courses as being taught by mathematics teachers. This is not correct. These courses require a very special person. Alternative or remedial courses are often taught by the least experienced math teachers or by misassigned nonmathematics teachers. This would guarantee failure for the courses. The teacher for these courses could logically be one of two basic types. One would be an experienced mathematics teacher with a strong problem solving emphasis and a good ability to write clearly. Since this type of teacher is in high demand for other courses, such assignment will require a strong commitment from the administration of the school to the success of the course.

A second option would be the assignment of a science or industrial arts teacher with a strong math background to teach the courses. This could present some difficulties. The technical mathematics course can only be looked on as a math course. The vocational math course, however, depending on the phrasing of the course description and syllabus, could be regarded as either an industrial arts course or a math course. This has been determined by actual, if unofficial, inquiry with the Teacher Standards and Practices Commission. To apply for the two year mathematics requirement, the language must make it obvious that the course is a math course. This means that, according to TSPC, the teacher, if not a math teacher, should have the Basic Combined Mathematics endorsement. If a science or industrial arts teacher can be found who meets this requirement, as well as the strong math background and the ability to write well, that might well be the ideal teacher. However, it may be that a teacher can only be found who meets all requirements except the endorsement (many science and industrial arts teachers do not have the required mathematics methods courses needed to qualify). If that is the case, the task force recommends that a misassignment be made in the interests of better education, assuming no suitable mathematics teacher can be found.

One of the most critical considerations in staffing the technical mathematics course is the importance the task force has assigned to the technical writing and extended problem segments of the course. It is a vital part of the course, not just an add-on. The task force believes that the ability to interpret and follow instructions, plan and execute problem solving approaches, and to report what has been planned and done in an intelligible manner is at least as important as the use of mathematical techniques or operations. For the type of student we envision in this course, the ability to communicate is essential. Integrating it with the mathematics at this level is somewhat radical and unique, but we believe entirely reasonable. It does, however, make staffing the course more difficult. Finding teachers who meet all other criteria but this one negates the intent of the course. Given a choice of superior math competence or superior communication/teaching ability with a commitment to mathematics, the preference should go to superior communications skills.

Certification

The discussion of staffing for these courses has led to the somewhat radical position that nonmathematics certified personnel be allowed to teach them. Under current certification regulations, nonmathematics certified teachers may obtain a Basic Combined Mathematics endorsement, allowing them to teach mathematics at the level of Algebra I or lower. Presumably, at least the vocational mathematics course could be taught with such an endorsement. The technical mathematics course would require a math endorsement.

As we have observed, nonmathematics certified teachers may, in many cases, be desirable as teachers for these courses. It would be unreasonable for many of these teachers to attempt to get mathematics certification or even a combined basic endorsement. The amount of additional coursework required to teach one or two sections of a class is excessive. A possible alternative would be to explore with the Teacher Standards and Practices Commission the possibility of establishing a special endorsement for these two classes. Nonmath certified teachers could obtain this endorsement by completing an inservice program or a single course offered by mathematics education departments training them to teach this course. This would eliminate some of the potential problems with misassignments, while at the same time providing a means of insuring properly prepared teachers.

Vocational Courses Receiving Mathematics Credit

In the past, some attempts have been made to award mathematics credit for vocational or industrial arts classes. The argument used to justify it is that some math skills are used in the courses, so it is appropriate to give math credit. These courses have usually had so little math content in them that attempts to award mathematics credit are ludicrous. It is vital that these proposed course outlines not be used as "ammunition" to justify awarding math credit for such courses.

Dick Moon and Ada Fancher are currently directing a project funded by the Oregon Department of Education, entitled "Integrated Vocational Education and Mathematics Skills Models." This project is looking as existing vocational/industrial arts courses in the Corvallis district with a view toward identifying mathematics content contained in them. In those courses which have substantial mathematics content, one-half year of mathematics credit may be awarded for completion of two years of work. Qualification of courses for this credit is a fairly involved process. Such awarding of credit is logical. Less rigorous procedures and attempts to award credit for vocational courses because they "use" mathematics is inappropriate.

The proposed courses are mathematics courses first, not vocational. They use the subject matter and content of vocational subjects as "tools" to present and develop the mathematics skills. This does not change the fact that they are primarily concerned with mathematics, even though usage is emphasized. Any course which is primarily vocational is not consistent with our outlines.

VI. Recommendations

It is the consensus of the task force that vocational and technical mathematics courses offer an opportunity to provide an option for the second year of the two year mathematics requirement at the same time that they fill some holes that exist in the current mathematics curriculum. For this reason we feel that development of the courses should be pursued with vigor. The logical group to oversee the development of these courses is OMEC. As attempts are made to develop them, it is advantageous to disseminate this paper as widely as possible, in order to gather support and assistance for the task. Accordingly, the task force makes the following recommendations:

1. Submit copies of this paper to ERIC, LIRA, and other appropriate organizations.

2. Present the paper to TOTOM, Oregon Math Leaders, Oregon Education Research Association, American Education Research Association, American Vocational Association, National Council of Teachers of Mathematics, Confederation of Oregon School Administrators.

3. Pursue various means of developing and testing course materials, including a cooperative effort by mathematics education departments in the state universities, submission of outlines to publishers, NSF or NIE grants, or task groups formed through Oregon Math Leaders.

4. Develop inservice for teachers who will teach the classes. Such inservice may be supported by school districts, NSF or NIE grants, funding from private foundations or corporations.

5. Explore the possibility of special certification or endorsements with TSPC.

6. Pursue inclusion of some preparation for teaching the courses in programs at Oregon colleges and universities for mathematics, vocational, and science teachers.

7. Develop a task force (perhaps through TOTOM) to assist school districts with implementation.

VOCATIONAL-TECHNICAL MATHEMATICS

TEXTS AND SOURCES

As noted in the body of the paper, there are no existing texts which fit the outlined courses well. All lack either the approach suggested, certain content strands, or both. There are, however, some materials which would be useful in planning or designing the courses. Conversely, there are some materials that would sound useful by their title, but in fact are not appropriate for the kinds of courses envisioned. This listing of sources is annotated to give the reader an idea of the appropriateness of the various material.

Austin, Jacqueline and Margarita Isern. Technical Mathematics. Saunders College Publishing, 1983.
Text develops some topic strands, but not others. Emphasis is on drill, traditional presentation and development of topics. Exercises and problems not relevant.

Calter, Paul. Technical Mathematics. Prentice-Hall, 1983.
Many useful problems in the text for technical math. Leans a little too much toward traditional courses in presentation of material.

Delmar Publishers. Practical Problems in Mathematics for:
This series of books is by a variety of authors.
Ball, John. Mason. 1973.
D'Arcangelo, Benedict and Bartholomew. Plumbers and Pipe Fitters. 1973.
Garrard, Crawford and Frank Boyd. Electricians. 1980.
Hoffman, Edward. Machinists.
Huth, Harry. Carpenters. 1979.
Moore, George. Automotive Technicians. 1979.
Schabel, Adolph J. Printing Trades. 1956.
Schell, Frank. Metric System. 1980.
Schell, Frank and Bill Matlock. Welders. 1982.
Schumacker, Fred. Sheet Metal Technicians. 1973.
Sullivan, Richard. Electronics Technicians. 1982.
These books provide a tremendous number of problems and applications. Not only do they provide great sources for the proposed courses, but they would be great for all algebra teachers.

Gerardi, William and William Jones. Essential Mathematics. Harcourt Brace Jovanovich, 1983.
Mostly numerical computation and repetitive exercises.

Glenn, Euel and David Wiltsie. Success in Mathematics. Motivation Development, Inc., 1981.
No problems, mostly drill.

Hannon, Ralph H. Mathematics for Technical Careers. Charles E. Merrill, 1976.
This text emphasizes numerical drill. It has few problems of any kind. They are word problems of the kind usually found in Algebra I texts. In fact, it looks much like a traditional algebra text.

Keedy, Mervin, Stanley Smith, and Paul Anderson. Applying Mathematics. Addison-Wesley, 1983.
A few good problems.

Lyng, Merwin J., L. J. Meconi, and Earl J. Zwick. Career Mathematics: Industry and the Trades. Houghton Mifflin Co., 1977.
A few useful problems, but largely numerical drill.

Moore, Claude, Bennie Griffin, Edward Polhamus, Jr. Applied Mathematics for Technicians. Prentice Hall, 1982.
Algebra is done in too much depth, not enough is covered in geometry. Other topics are not covered at all. A few good problems.

National Council of Teachers of Mathematics. Applications in School Mathematics--1979 Yearbook.
Excellent source of suggestions that apply in most areas of the proposed courses.

National Council of Teachers of Mathematics. Teaching Statistics and Probability--1981 Yearbook.
Current ideas which lend themselves to inclusion in other topics, including application or extended problems.

Oregon Department of Education. Oregon Vo-Tech Mathematics Problem Sets. 1974.
A set of booklets, each dealing with mathematics in a vocational/industrial/technical area.
Agriculture
Aviation Mechanics
Auto Mechanics
Clerical
Construction
Diesel Mechanics
Drafting
Electronics
Fire Science
Food Processing
Forestry
Industrial, Electrical, Hydraulics
Industrial Mechanics
Machine Tools
Marketing
Nursing
Police Science
Real Estate
Waste Water Technology
Wood Products
Welding
These booklets present a wealth of useful problems for inclusion in either course. Some are short problems, some lend themselves to expansion into the suggested extended problems.

Pachucki, Chester. Mathematics for Industrial Technicians. Prentice-Hall, 1974.
Essentially a traditional approach.

Peterson, John and William deKryger. Mathematics for the Automotive Trade. Southwestern Publishing Co., 1983.
Practical problems, but much of the book is devoted to drill, especially in computation.

Pouler, Wilfred B. Print Reading for the Machine Trades. Southwestern Publishing Co., 1984.
Highly technical and job oriented textbook. It provides some useful exercises for visualization.

RECSAM Participants. Mathematics for Vocational Students. RECSAM Conference, April 12-June 24, 1976.
A few useful problems. Little content as envisioned by the task force. Could be useful as source of extended problems.

Saunders, Hal. When Are We Ever Gonna Have To Use This? Dale Seymour Publications, 1981.
An excellent source of problems.

Shaw, Bryce and Robert Kramer. Fundamentals of Mathematics--Skills and Applications. Houghton Mifflin, 1982.
Mostly drill, not application.

Singer, Bertrand B. Mathematics at Work: Algebra. McGraw Hill, 1977.
Presentation of material not appropriate, but useful problems.

Singer, Bertrand B. Mathematics at Work: Decimals. McGraw Hill, 1977.
Less useful than the algebra book, but still a good source of problems.

Smith, Robert D. Vocational Technical Mathematics. Delmar Publishers, Inc., 1983.
This text covers some, but not all, topic strands. Exercises are mostly word problems and exercises as would be seen in standard algebra books. Presentation of geometry is a little too formal. Algebra presentation is far too formal for the vocational math course.

Smith, Robert D. Applied General Mathematics. Delmar Publishers, Inc., 1982.
Too much numerical computation, some good problems.

Wiltsie, David. Vocational Mathematics Series.
Book 1 Fractions, Decimals, Percents
Book 2 Metric and U.S. Systems of Measurement
Book 3 Practical Uses for Formulas and Equations
Book 4 Mathematics of Geometric Shapes and Objects
Book 5 Machine Mathematics (Gears, Pulleys, and Levers)
Book 6 Carpentry Mathematics
Book 7 Working with Electric Circuits

Book 8 Mathematics for Shoppers and Salespeople
Book 9 Mathematics for Employment and Business
Motivation Development, Inc., 1982.

These books come the closest to the approach and content envisioned by the task force. Some topic strands are not covered. Books 1-4 are too traditional/conventional in their presentation of material. Many useful problems.